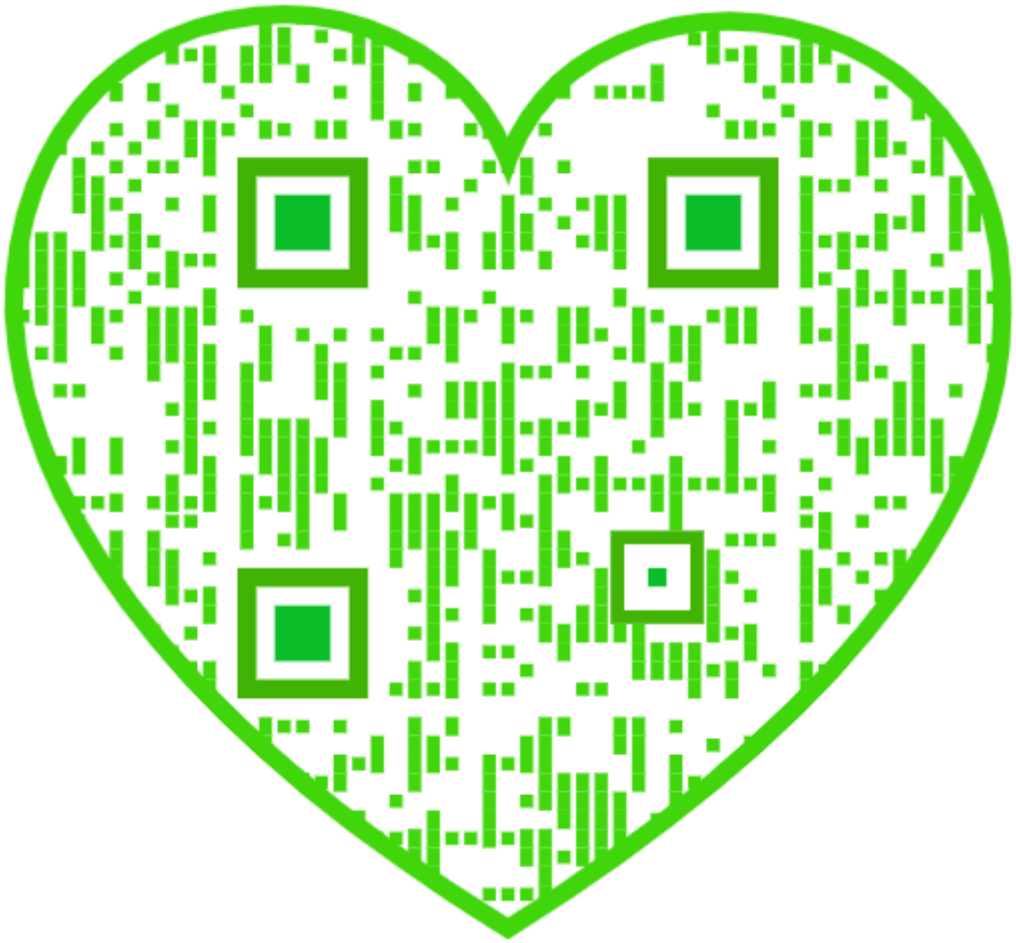

NUMBERS STEP BY STEP

AN

Elementary Arithmetic

By

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PREFACE

The features that distinguish this book from other elementary arithmetics, and which in the experience of the author make it more teachable, are:

FIRST. The general arrangement of the book is topical in the truest sense. The difficulties are arranged in what is believed to be the order of "least resistance." Each difficulty is then made the subject of a lesson.

SECOND. The fact, that all concrete problems may be solved by counting instead of by computation, is used to determine, in a given problem, which of the computative processes is to be employed.

THIRD. The multiplication table is used for reference purposes before it is learned, thus furnishing an adequate motive for the stupendous task of memorizing it.

FOURTH. Exponents are used in adding a column of figures, in order that children of the second grade, who know the addition table without knowing its extensions, may solve concrete problems requiring the addition of four or five numbers. Later, when the extensions of the addition table have been learned, the exponents will not be needed unless the columns are very long.

FIFTH. Artificial units are introduced with the utmost care. Such attributes as weight, capacity, and time, do not naturally suggest plurality. The assumption that number can be artificially applied to them by young children without careful instruction is erroneous. The practice based on this assumption is sure to dull the number perception of pupils.

SIXTH. The computative process of long division is so treated that its three difficulties are met and mastered one at a time.

FRANK RIGLER

January, 1913.

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INTRODUCTION

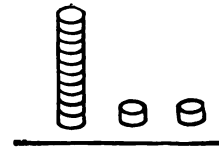
This book assumes that pupils who begin to use it have learned: (1) to read and write numbers to 400; (2) to solve problems by counting, each of the counters used representing one of the units about which the problem concerns itself; (3) to express numerically a solution just performed by counting; (4) to imagine the counting instead of performing it, in order to get the numerical expression, and to consult the addition or multiplication table for the result; (5) the forty-five combinations of the addition table; (6) to apply both real and imagined counting to the following artificial units: pint, quart, gallon, peck, bushel, ounce, pound.

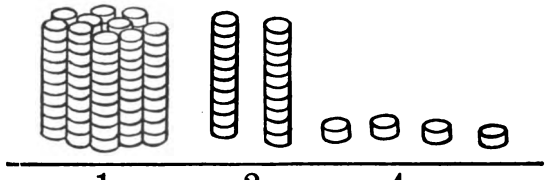
If, as will often be the case, pupils have not received this preliminary training, two courses are open, as follows:

FIRST. Use the material furnished by this book in the order given, but base the instruction upon such number knowledge as the pupil possesses. Probably the chief thing he lacks will be the ability to imagine a solution by counting, and from the imagined mode of counting to determine the operation to be performed. Very likely, if any problem-solving has been attempted at all, verbal rather than number relations have been depended on to give the mode of computation. If the teacher, either because of preference for this method, or for any other reason, desires to continue its use, she will find the material of this book well adapted to her purpose. If, however, her interest has been so far aroused that she wishes to base her instruction on those principles upon which the author has based his book, then she may proceed as follows:

SECOND: (1) *To read and write numbers to 400.*

The instruction already received by the children may need to be supplemented by the objective representation of numbers, the counters used being grouped according to the decimal system of notation. Thus 12 is represented by a pile of ten counters, and two single ones to the right of the pile. 124 is represented by ten piles of ten each grouped together, to the right of which are two piles of ten each, while further to the right are four single ones.





(2) *To solve problems by counting, using counters instead of the objects themselves.*

The counter is the universal symbol of all concrete units. To the child it may stand for a pencil, a horse, a bird, a book, or any individual unit whatever. Later, when artificial pluralities are dealt with, it may represent a pint, a pound, an hour, or any other artificial unit.

An ordinary gun wad makes a very suitable counter. Its size, shape, and noiselessness all recommend it, though, of course, any counter may be used without seriously affecting the character of the work.

The mental connection to be established is to have *the conditions of the problem suggest the mode of counting* necessary for the objective solution of the problem.

(a) *Additive Counting.*

Announce orally this problem: There were 5 books on the table. Mary moved 2 more books from the shelf to the table. How many books were then on the table?

Have it repeated by several different pupils, in order that its conditions may be familiar to all the pupils of the class.

Let each pupil take 5 counters from his supply to represent the 5 books already on the table.

Then let each take 2 more counters from his supply, to represent the 2 books Mary moved from the shelf to the table.

Then, counting all the counters together, he finds there are 7 books on the table.

After this has been done, let some pupil tell the story about as follows: "There were 5 books on the table. Mary moved 2 more books from the shelf to the table. There were then 7 books on the table." No fixed form should be insisted on, because much criticism of the manner of telling will divert the pupil's attention from number relations to word sequences.

After the story has been told it is well to rehearse the mode of counting, somewhat as follows:

Teacher. "What did you do first?"

Pupil. "I took 5 counters from the supply."

Teacher. "What did you do next?"

Pupil. "I took 2 more counters from the supply."

Teacher. "What next?"

Pupil. "I counted them together and found 7 counters or 7 books on the table."

This drill not only helps to fix the steps of the counting in mind, but also is of great assistance later in substituting imagined counting for actual counting.

(b) *Subtractive Counting.*

PROBLEM: John had 7 marbles in a bag. He gave 4 of them to George. How many marbles were left in the bag?

The conditions are fixed in mind by repetition.

Then pupils count 7 counters from the supply, to represent the 7 marbles in the bag.

They then count 4 counters from the 7 (not from the supply as in additive counting), to represent the 4 marbles given to John.

Counting what are left, they find 3 marbles still in the bag.

The story should then be told and the mode of counting rehearsed, as in additive counting.

(c) *Partitive Counting.*

PROBLEM: I have 12 pencils. I wish to give them all to Thomas, Martha, and Edna, so that Thomas shall have just as many as Martha, and Martha shall have just as many as Edna. How many will each child have?

The conditions of the problem are fixed by repetition.

Then pupils count 12 counters from the supply, to represent the 12 pencils.

They then put one in one place for Thomas; one in another place for Martha; and one in a third place for Edna. Each child has now been given one pencil.

This counting (one at a time to each) is repeated, and each child then has 2 pencils.

The mode of counting (one at a time) is continued until the pencils are all gone, when it is found that each child has received 4.

The story should then be told and the mode of counting rehearsed.

As soon as the conditions of the problem promptly suggest the mode of counting, *i.e.*, whether it is *additive, subtractive,* or *partitive,* the mental connections sought have been established, and pupils are ready for the next step.

(3) *Numerical expressions should be found for the solution of problems by counting.*

To have the mode of counting suggest an appropriate numerical expression is the mental connection sought.

(a) *The Additive Expression.*

Suppose a problem, say the one given above under additive counting, which requires 5 counters and 2 counters to be moved from the supply and counted together. After pupils have done the counting and announced the result, the teacher

by appropriate questions makes it plain to the pupil that the numerical expression $\begin{array}{r} 5 \\ +2 \\ \hline 7 \end{array}$ records the story, 5 standing for

the five books already on the table; 2 for the two books afterwards moved near them; + for the act of counting them together; and 7 for the result obtained.

(b) *The Subtractive Expression.*

After a problem requiring subtractive counting (say the one given above) has been solved with counters and the result announced, let the teacher show that the story may be written thus: $\begin{array}{r} 7 \\ -3 \\ \hline 4 \end{array}$ Care should be taken to point out

the function of the 7, the 3, the −, and the 4.

(c) *The Partitive Expression.*

A problem in partitive counting (say the one given above) should be solved by counting; then recorded thus: $\frac{1}{3}$ of $12=4$, in which 12 shows the number of pencils, $\frac{1}{\ }$ that they were counted "one at a time," $\frac{}{3}$ into 3 places (*i.e.*, in this problem, to 3 children), and 4 shows how many were put in each place, (*i.e.*, given to each child).

(d) *The Multiplicative Expression.*

Whenever in additive counting the numbers added are of the same size, the multiplicative expression may be used. Thus, suppose a problem giving the expression: $\begin{array}{r} 4 \\ 4 \\ +4 \\ \hline 12 \end{array}$ By appropriate

questions show that 4 is written three times, which may be expressed 3×4, and the result of the counting may be written $=12$, making the whole expression $3\times4=12$.

(e) *Measurement, the numerical expression of a special case of subtractive counting.*

Teacher announces the following problem: "I have 12 roses and wish to put them into vases, 4 in each vase. How many vases will I need?"

After the story has been repeated and its conditions fixed in the minds of the pupils, each of the class takes 12 counters representing the 12 roses. Then, as a result of appropriate questions or directions, 4 counters are taken from the 12, indicating the roses to be put into the first vase; 4 more counters are taken away for the second vase; and the last 4 for the third vase. The counting shows that 3 vases are needed.

By questioning the teacher gets this record,

$$\begin{array}{r} 12 \\ -4 \\ -4 \\ -4 \\ \hline 0 \end{array}$$

and then tells the class that $4\overline{)12}$ is shorter, and means that 4 can be taken from 12, three times.

$$3$$

The use of the counters should be continued until the mode of counting employed at once suggests its proper numerical expression. In the language employed by the teacher to the pupil, the latter should be able:

First, to *tell* the story, (*i.e.*, repeat the problem).

Second, to *count* the story, (*i.e.*, perform the mode of counting suggested by the conditions of the problem).

Third, to *write* the story, (*i.e.*, write the numerical expression suggested by the mode of counting employed).

When the pupil can do these three things promptly in the order named, the mental connections sought have been established, and he is ready to take the next step.

(4) *Getting the numerical expression by imagining the mode of counting, and getting the result from the addition table or the multiplication table.*

On the blackboard should be placed the addition and multiplication tables as given on page 360 of this book.

A problem being given, the pupils are led by suitable questions to think how they would count, (see rehearsal of counting, pages 10 and 11), then they are told to write the problem that far, *i.e.*, they are able at the suggestion of the imagined mode of counting to write the numerical expression without the result.

They should then be taught how to consult the tables for results.

The method of consulting the addition table is obvious. The use of the multiplication table is shown in the explanatory exercises on pages 21, 25, and 30 of this book.

(5) *Memorizing the addition table.*

Probably the pupil has already been taught this, since most of the beginning methods in arithmetic make their first aim the teaching of certain "number facts" or "combinations." These words are synonyms for the tables. When the forty-five facts of the addition table have been thoroughly learned, it is no longer necessary to keep that table on the blackboard.

(6) *The use of artificial pluralities.*

Although certain artificial units are presumed to have been used by the pupil before he takes up this book, nevertheless, their use is explained here as if all of his prior work had been with natural units only. This is review work for some who will use the book, and makes it possible for all pupils to begin the book at the middle of their second school year.

For a more detailed account of the exercises described in this introduction, the teacher is referred to a primer of number written by the author of this book, and entitled "Measured Plurality."

The Birthday Week

Monday's child is fair in the face;
Tuesday's child is full of grace;
Wednesday's child is the child of woe;
Thursday's child has far to go;
Friday's child is loving and giving;
Saturday's child works hard for a living;
But the child that is born on the Sabbath day
Is lucky and bonny, and wise and gay.

NUMBERS STEP BY STEP

CHAPTER I

STEPS OF SECOND YEAR

LESSON ONE

HOW TO GET THE CONDITIONS OF A WRITTEN PROBLEM

I. NOTE TO TEACHER. Heretofore all concrete problems have been announced orally by the teacher. Now they are to be announced in writing on the blackboard or in print in this book. It is much harder to get the meaning of what we read than it is to understand what is told us. A large portion of the failures to solve concrete problems is due to the fact that pupils do not understand their conditions from reading them.

To correct this, pupils must be required to read problems aloud. They should then be questioned to see that they have clearly in mind the conditions described in the problem.

In this connection it is not necessary to ascertain whether or not the pupil can solve the problem, but he should understand its conditions.

II. DRILL, by having pupils read and talk about problems in this book for a few days. When they see clearly what these problems are about, then it is time to commence to solve problems that they have read silently and understood without help.

UNCLASSIFIED PROBLEMS

1. Mary had 9 paper dolls; Ellen had 8. How many paper dolls had both girls?

2. John bought 16 sticks of candy. He ate 7 of them, and gave away the others. How many did he give away?

3. 14 eggs were put under a sitting hen. 9 chickens were hatched. How many of the eggs were bad?

4. Willie has 4 white marbles and 8 brown ones. How many marbles has Willie?

5. Bessie spelled 15 words. 7 of them were wrong. How many were right?

6. On one dogwood branch there were 7 blossoms. On another branch there were 6 blossoms. How many blossoms were on both branches?

7. In a bouquet were 9 pink roses and 6 yellow roses. How many roses of both kinds were in the bouquet?

8. A little girl picked 14 bunches of pansies. She sold all of them but 5 bunches. How many bunches did she sell?

9. There were 12 people in a street car. 7 of them left the car at Main street. How many remained in the car?

10. Three boys went fishing. The first boy caught 5 fish. The second boy caught 6 fish. The third boy did not catch any. How many fish did they all catch?

LESSON TWO

HOW TO USE THE MULTIPLICATION TABLE IN SOLVING CONCRETE PROBLEMS IN MULTIPLICATION

I. EXPLANATION. (a) *Preparation.* Have the table on page 360 of this book placed on the blackboard.

(b) *Problem.* A tailor made 8 vests. On each vest he placed 6 buttons. How many buttons did he use?

This problem is to be read, either from the blackboard or book, by one or more pupils, until its conditions are familiar to the class.

(c) *Exercise.*

Teacher. "What would you do first, Jane?"

Jane. "Count out six buttons from the supply."

Teacher. "How many times would you do this, May?"

May. "Eight times."

Teacher. "Who can write the story that far?"

After hands are shown, a pupil is sent to the board. He writes, $8\times6=$

Teacher. "You may all write the story that far."

Pupils do so.

Teacher. "Let us finish the story from the table instead of by counting. John, take a pointer; move it downward from the top of the second column, until you come to $8\times$. Then keep your pointer at that place."

John does so.

Teacher. "William, you may take another pointer, place it at the left end of the top row, and move it towards the right until you come to 6. Keep it at that place."

"Now, while John moves his pointer to the right, William may move his downward until the pointers meet."

The boys do so.

Teacher. "At what number do they meet, Lucy?"

Lucy. "Forty-eight."

Teacher. "Yes; you may finish the story by writing 48; thus, $8 \times 6 = 48$."

Pupils do so.

Teacher. "Samuel may tell me the story."

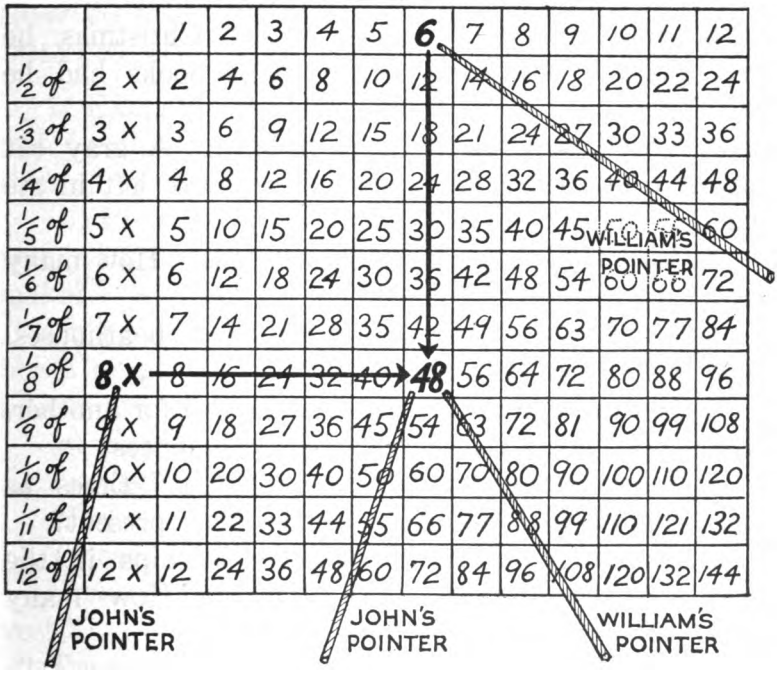
Samuel says something equivalent to the following: (No set form is insisted on.)

"A tailor made 8 vests. On each vest he placed 6 buttons. He used 48 buttons."

II. DRILL. Finish the following from the table:

$3 \times 4 =$	$5 \times 7 =$	$9 \times 9 =$
$4 \times 8 =$	$3 \times 5 =$	$2 \times 6 =$
$9 \times 3 =$	$2 \times 9 =$	$5 \times 4 =$
$7 \times 5 =$	$6 \times 7 =$	$8 \times 9 =$
$6 \times 2 =$	$4 \times 6 =$	$10 \times 5 =$
$3 \times 9 =$	$8 \times 8 =$	$3 \times 7 =$
$2 \times 5 =$	$7 \times 9 =$	$4 \times 9 =$
$5 \times 8 =$	$6 \times 5 =$	$6 \times 8 =$
$8 \times 3 =$	$10 \times 4 =$	$11 \times 10 =$
$11 \times 11 =$	$12 \times 5 =$	$7 \times 8 =$

ILLUSTRATION



		1	2	3	4	5	6	7	8	9	10	11	12
½ of	2 ×	2	4	6	8	10	12	14	16	18	20	22	24
⅓ of	3 ×	3	6	9	12	15	18	21	24	27	30	33	36
¼ of	4 ×	4	8	12	16	20	24	28	32	36	40	44	48
⅕ of	5 ×	5	10	15	20	25	30	35	40	45	50	55	60
⅙ of	6 ×	6	12	18	24	30	36	42	48	54	60	66	72
⅐ of	7 ×	7	14	21	28	35	42	49	56	63	70	77	84
⅛ of	8 ×	8	16	24	32	40	48	56	64	72	80	88	96
⅑ of	9 ×	9	18	27	36	45	54	63	72	81	90	99	108
1/10 of	10 ×	10	20	30	40	50	60	70	80	90	100	110	120
1/11 of	11 ×	11	22	33	44	55	66	77	88	99	110	121	132
1/12 of	12 ×	12	24	36	48	60	72	84	96	108	120	132	144

UNCLASSIFIED PROBLEMS

1. Each of 6 plates contained 9 candy eggs. How many candy eggs were on all the plates?

2. Edna had 7 photographs taken. 3 of them were poor. How many were good?

3. In Jane's garden are 9 rows of lily bulbs, with 8 lily bulbs in each row. How many lily bulbs are in the garden?

4. If there are 9 panes in one window and 4 in another, how many panes are there in both?

5. John had 8 old books. On Christmas he was given 3 new ones. How many books had he then?

6. There were 15 mice in a barn. A gray cat caught 6 of them. How many mice were left in the barn?

7. Each of 4 boys made 6 snowballs. How many snowballs did they all make?

8. There are 8 boats in each of 4 boathouses. How many boats altogether?

9. It took 7 posts for one fence and 9 for another. How many posts did it take for both fences?

10. There were 5 pupils absent, and 7 times as many present. How many pupils were present?

11. In the morning I counted 15 ships in the river. At noon there were only 7. How many ships had sailed away?

12. Mary has 11 dolls. They are all paper dolls but 4. How many paper dolls has she?

13. There are 5 rooms in a new house. It will take 4 rugs for each room. How many rugs will be needed in the house?

14. George walked 3 blocks and rode 9. How far did he go?

15. On a track there are 5 cars. Each car has 8 wheels. How many wheels have all the cars?

LESSON THREE

HOW TO CONSULT THE MULTIPLICATION TABLE IN SOLVING CONCRETE PROBLEMS IN MEASUREMENT

I. EXPLANATION. (a) *Preparation.* The instruction in this lesson is based on the same table as that of the previous lesson.

(b) *Problem.* A tailor placed 6 buttons on each vest. He used 48 buttons. How many vests did he make?

This problem is to be read, either from the blackboard or book, by one or more pupils, until its conditions are familiar to the class.

(c) *Exercise.*

Teacher. "What would you do first, Mary?"

Mary. "Count 48 buttons from the supply."

Teacher. "What next, Samuel?"

Samuel. "Take 6 buttons from 48, for the first vest."

Teacher. "Then what?"

Samuel. "Take 6 more buttons for the next vest."

Teacher. "Then what?"

Samuel. "Keep doing this until all the 48 buttons are used."

Teacher. "Who can write the story that far?"

After hands are shown, a pupil is sent to the board. He writes, $6\overline{)48}$

Teacher. "You may all write the story that far."

Pupils do so.

Teacher. "Let us try to finish the story from the table instead of by counting.

"Sarah, take a pointer; place it at the left end of the top row.

"Move it towards the right until you come to 6.

"Move it downward from 6 to 48.

"Move it to the left from 48 to the second column.

"What have you there?"

Sarah. "Eight times."

Teacher. "You may finish the story by writing $8\times$, thus, $\frac{6)48}{8\times}$"

Pupils do so.

Teacher. "Richard, you may tell the story."

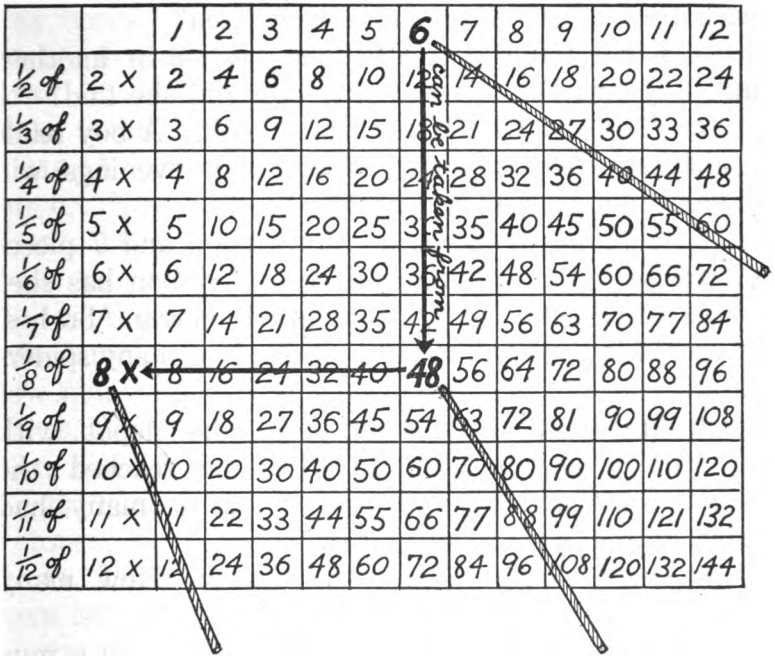
Richard says something equivalent to the following: (No set form is insisted on.)

"A tailor placed 6 buttons on each vest until he had used 48 buttons. He made 8 vests."

II. DRILL. Finish the following from the table:

2)14	8) 56	10)100	7)35	4)24
5)30	3) 24	5) 45	8)72	8)24
9)63	6) 18	9) 72	2)16	11)77
7)84	12)108	4) 28	3)30	12)60
4)32	12) 48	2) 8	5)40	7)49
11)33	3) 36	6) 48	6)12	9)27

ILLUSTRATION



		1	2	3	4	5	6	7	8	9	10	11	12
½ of	2 X	2	4	6	8	10	12	14	16	18	20	22	24
⅓ of	3 X	3	6	9	12	15	18	21	24	27	30	33	36
¼ of	4 X	4	8	12	16	20	24	28	32	36	40	44	48
⅕ of	5 X	5	10	15	20	25	30	35	40	45	50	55	60
⅙ of	6 X	6	12	18	24	30	36	42	48	54	60	66	72
⅐ of	7 X	7	14	21	28	35	42	49	56	63	70	77	84
⅛ of	8 X	8	16	24	32	40	48	56	64	72	80	88	96
⅑ of	9 X	9	18	27	36	45	54	63	72	81	90	99	108
1/10 of	10 X	10	20	30	40	50	60	70	80	90	100	110	120
1/11 of	11 X	11	22	33	44	55	66	77	88	99	110	121	132
1/12 of	12 X	12	24	36	48	60	72	84	96	108	120	132	144

UNCLASSIFIED PROBLEMS

1. The following combinations in addition and subtraction should be placed on the blackboard and thoroughly drilled upon every day.

3	2	3	5	7	4
+7	+9	+8	+6	+9	+6

10−3	11−8	16−9	10−4	16−7	11−9
11−2	10−7	11−6	11−3	10−6	11−5

2. 18 mice were caught in a store one night, 6 in each trap. How many traps were used?

3. Ethel found 4 eggs in one nest, 2 in another, and 3 in another. How many eggs did she find?

4. There are 24 chapters in a book. A boy reads 3 chapters each evening. How many evenings will it take him to read the book?

5. Alice has 8 pieces of blue ribbon and 7 pieces of red ribbon. How many pieces of ribbon has she?

6. 7 families of spiders live in our currant bushes. If there are 8 spiders in each family, how many spiders in all?

7. An expressman started for the depot with 10 trunks on his wagon. When he reached the depot, he found he had only 8. How many had fallen off?

8. Henry learns 4 lessons a day. How many lessons will he learn in 6 days?

9. On a checkerboard are 64 squares, 8 in a row. How many rows are there?

10. John shot a gun 17 times, and hit the target 8 times. How many times did he miss?

11. A newsboy sold 9 morning papers and 8 evening papers. How many papers did he sell?

12. If each cage holds 6 birds, how many birds will 7 cages hold?

13. There were 36 pupils in a room. The teacher formed classes and put 12 pupils in each class. How many classes did she form?

14. There are 12 men in a band. All of them play horns but 3. How many play horns?

15. A carpenter built 7 houses. He put 9 windows in each house. How many windows did he make in all?

16. A lady planted 14 rosebushes. They all died but 5. How many died?

17. There are 9 birds on a fence, 5 on a tree, and 1 on the ground. How many birds altogether?

18. A gardener had 60 onions, and made them into bunches. If he put 12 onions in each bunch, how many bunches did he make?

19. 3 little girls had 10 roses apiece. How many roses had they all?

20. I gave 42 tops to some boys, giving each boy 7 tops. How many boys were there?

21. There were 15 birds on a fence. How many were left after 8 flew away?

22. A girl has seven 10-cent pieces. How much money has she?

23. A girl has 9 paper dolls, and 3 dresses for each. How many doll's dresses has she?

24. A girl picked 40 flowers and put them into 5 bunches. The bunches all had the same number of flowers. How many flowers were in each bunch?

25. Two girls have 16 cents between them; one has 7 cents. How many cents has the other?

26. A boy bought 17 newspapers and sold 9. How many had he left?

LESSON FOUR

HOW TO CONSULT THE MULTIPLICATION TABLE IN SOLVING CONCRETE PROBLEMS IN PARTITION

I. EXPLANATION. (a) *Preparation.* The instruction in this lesson is based on the same table as that of the previous lesson.

(b) *Problem.* A tailor used 48 buttons in making 8 vests. All the vests had the same number of buttons. How many buttons did he sew on each vest?

This problem is to be read, either from the book or blackboard, by one or more pupils, until its conditions are familiar to the class.

(c) *Exercise.*

Teacher. "What would you do first, George?"

George. "Count 48 buttons from the supply."

Teacher. "What next, Charles?"

Charles. "Count out the buttons 'one at a time' in 8 places, until all are placed."

Teacher. "Who can write the story that far?"

After hands are shown, a pupil is sent to the board. He writes, $\frac{1}{8}$ of 48 =

Teacher. "You may all write the story that far."

Pupils do so.

Teacher. "Let us try to finish the story from the table instead of by counting. Susan, take a pointer and place it at the top of the first column.

"Move it downward until you come to $\frac{1}{8}$.

"Move it to the right to 48.

"Move it upward from 48 to the top of that column.

"What do you find there?"

Susan. "Six."

Teacher. "You may finish the story by writing 6, thus, $\frac{1}{8}$ of 48 = 6."

Pupils do so.

Teacher. "Susan, you may tell the story."

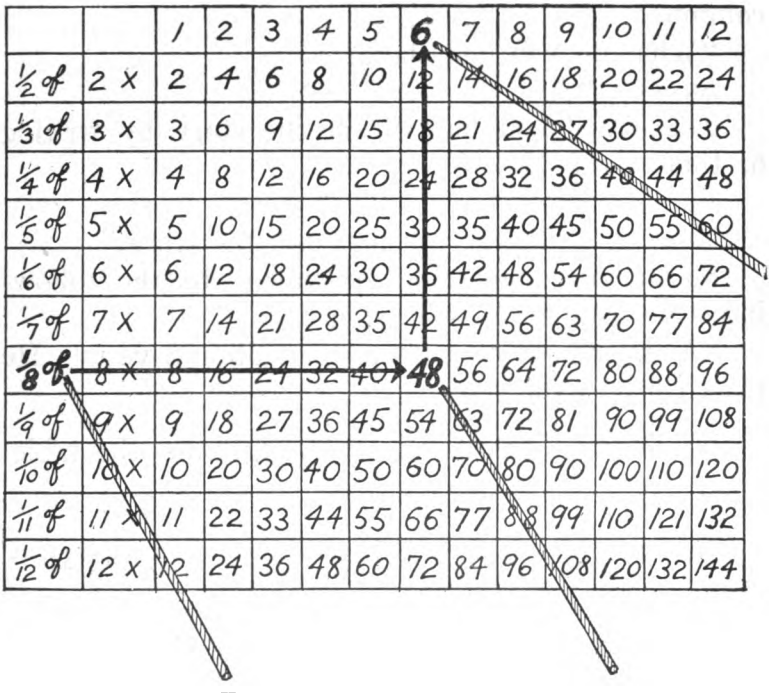
Susan says something equivalent to the following: (No fixed form is insisted on.)

"A tailor used 48 buttons in making 8 vests. He placed 6 on each vest."

II. DRILL. Finish the following from the table:

$\frac{1}{2}$ of 8 =	$\frac{1}{9}$ of 45 =	$\frac{1}{3}$ of 9 =
$\frac{1}{4}$ of 24 =	$\frac{1}{8}$ of 56 =	$\frac{1}{4}$ of 16 =
$\frac{1}{9}$ of 27 =	$\frac{1}{2}$ of 24 =	$\frac{1}{5}$ of 40 =
$\frac{1}{12}$ of 48 =	$\frac{1}{7}$ of 63 =	$\frac{1}{6}$ of 54 =
$\frac{1}{3}$ of 36 =	$\frac{1}{5}$ of 35 =	$\frac{1}{9}$ of 54 =
$\frac{1}{10}$ of 60 =	$\frac{1}{3}$ of 33 =	$\frac{1}{8}$ of 40 =
$\frac{1}{5}$ of 15 =	$\frac{1}{11}$ of 55 =	$\frac{1}{7}$ of 49 =
$\frac{1}{8}$ of 24 =	$\frac{1}{6}$ of 48 =	$\frac{1}{2}$ of 4 =
$\frac{1}{11}$ of 110 =	$\frac{1}{4}$ of 12 =	$\frac{1}{12}$ of 132 =
$\frac{1}{7}$ of 42 =	$\frac{1}{12}$ of 72 =	$\frac{1}{5}$ of 45 =
$\frac{1}{8}$ of 72 =	$\frac{1}{7}$ of 56 =	$\frac{1}{3}$ of 12 =

ILLUSTRATION



		1	2	3	4	5	6	7	8	9	10	11	12
½ of	2 ×	2	4	6	8	10	12	14	16	18	20	22	24
⅓ of	3 ×	3	6	9	12	15	18	21	24	27	30	33	36
¼ of	4 ×	4	8	12	16	20	24	28	32	36	40	44	48
⅕ of	5 ×	5	10	15	20	25	30	35	40	45	50	55	60
⅙ of	6 ×	6	12	18	24	30	36	42	48	54	60	66	72
⅐ of	7 ×	7	14	21	28	35	42	49	56	63	70	77	84
⅛ of	8 ×	8	16	24	32	40	48	56	64	72	80	88	96
⅑ of	9 ×	9	18	27	36	45	54	63	72	81	90	99	108
⅒ of	10 ×	10	20	30	40	50	60	70	80	90	100	110	120
1/11 of	11 ×	11	22	33	44	55	66	77	88	99	110	121	132
1/12 of	12 ×	12	24	36	48	60	72	84	96	108	120	132	144

UNCLASSIFIED PROBLEMS

1. Drill each day on the following combinations:

3	3	5	6	6	6	8
+5	+6	+7	+8	+9	+7	+8
8	9	12	14	15	13	16
−3	−3	−5	−8	−6	−6	−8
8	9	12	14	15	13	12
−5	−6	−7	−6	−9	−7	−3

These should be placed on the board in a different order each day and rapidly recited by individual pupils, never in concert.

2. A man sold 27 pigeons to 9 boys. Each boy bought the same number. How many pigeons did each boy buy?

3. Harold made 4 kites. He used altogether 32 tacks. How many tacks were used for each kite?

4. A postman took 9 letters from each of 4 boxes. How many letters did he take in all?

5. A lady made 21 aprons for her 3 little girls. How many aprons could each girl have?

6. Clyde used 5 of the square tablets in his model box to make a rosette. How many tablets would it take to make 3 rosettes?

7. May bought 24 pencils, 6 in a box. How many boxes of pencils did she buy?

8. From my window I can see 8 poplar trees and 7 elm trees. How many trees can I see?

9. 24 clocks filled 8 equal boxes. How many clocks were put into each box?

10. A lady baked 18 pies and put them in the pantry, 6 on a shelf. How many shelves did she use?

11. A man had 56 pencils which he divided among some children. He gave each child 8 pencils. How many children were there?

12. A lady sewed 35 buttons on some waists. She put 5 buttons on each waist. How many waists were there?

13. Our teacher gave a drill in the hall. She had 5 rows of pupils with 8 in each row. How many pupils were in the drill?

14. The teacher called 4 girls to pass around 24 books. How many books did she give them apiece, if they all received the same number?

15. A boy has 11 rabbits. 6 of them are white and the others are gray. How many gray rabbits are there?

16. 20 apples were used in making 5 pies of the same size. How many apples was that for each pie?

17. At a store were 21 boxes of strawberries. Some customers bought 3 boxes each. How many customers were supplied, if all the strawberries were sold?

18. 4 boys had altogether 28 ducks. All of the boys had the same number. How many ducks did the boys have apiece?

19. Nellie had 4 pieces of red chalk, 5 pieces of blue chalk, and 8 pieces of white chalk. How many pieces of chalk did she have?

20. John made 5 piles of his blocks. There were 9 blocks in each pile. How many blocks had he?

21. On a shelf were 14 books. 9 of them had cloth covers. The others had paper covers. How many had paper covers?

22. 3 boys had altogether 12 flags. If they had the same number apiece, how many flags had each boy?

LESSON FIVE

HOW TO ADD A COLUMN, USING EXPONENTS

I. EXPLANATION. Suppose this column is to be added; $4 + 5 + 6 + 7 + 5$.

4	4	4	4
5	5	5^{3}	5^{3}
6	6	6	6
7	7^{2}	7^{2}	7^{2}
+5	+5	+5	+5
			27

Commencing at the bottom, 5 and 7 are 12. If we drop the one ten, we may keep the 2 by writing it as an exponent of 7, making the column appear as in its second position.

Commencing with the exponent 2, 2 and 6 are 8 (no ten to drop), 8 and 5 are 13. Drop the one ten and keep the 3 by writing it as an exponent of 5, making the column appear as in its third position.

Commencing with the exponent 3, 3 and 4 are 7. Write 7 below the line. Count the exponents to find how many tens were dropped. Finding that there were 2, write the 2 tens beside the seven, making the column appear as in its fourth position.

II. DRILL. Add these columns, using exponents:

(1)	(2)	(3)	(4)	(5)	(6)	(7)
4	3	8	5	2	4	5
3	4	0	8	5	9	8
6	6	7	4	6	3	6
5	8	5	6	5	8	4
+8	+2	+6	+1	+4	+7	+5

(8)	(9)	(10)	(11)	(12)	(13)	(14)
5	4	5	9	2	5	5
4	2	8	5	0	9	9
2	0	6	6	3	8	4
7	5	2	4	6	7	3
8	6	3	1	1	6	7
+2	+7	+4	+5	+2	+8	+2

UNCLASSIFIED PROBLEMS

1. Drill every day on the following combinations in addition and subtraction:

4	5	4	7	2	3	4
+7	+8	+9	+8	+8	+9	+8
11	13	13	15	10	12	12
−4	−5	−4	−7	−2	−3	−4
11	13	13	15	10	12	12
−7	−8	−9	−8	−8	−9	−8

2. 45 books belonged to 9 pupils. All of the pupils had the same number of books. How many books had each pupil?

3. A party of girls made 48 soap bubbles. They made 8 bubbles apiece. How many girls were in the party?

4. James had 4 tops, and bought 3 more. William had 6 tops, and bought 5 more. How many tops did both boys then have?

5. A blacksmith shod 7 horses in a day. He put 4 horseshoes on each horse. How many horseshoes did he fit in a day?

6. 24 roses were placed in vases. There were 8 roses in each vase. How many vases were used?

7. With the teacher's help, add these columns:

5	7	6	4
2	5	2	3
6	9	7	7
8	3	5	5
+4	+8	+8	+6

8. A woman sewed 42 buttons on 7 pairs of gloves. How many buttons was that for each pair?

9. There were 14 ripe cherries on Willie's cherry tree. The birds ate 6. How many were left?

10. A school has 24 boys and 31 girls. How many pupils in the school?

11. The teacher wrote 32 words on the board. She put the words in 4 equal rows. How many words were in each row?

12. George showed his little brother some picture books. There were 6 pictures in each book, and 24 pictures altogether. How many picture books were there?

13. In a street car there were 7 men, 6 women, and 9 children. How many people were in the car?

14. We go to school 5 days a week. We have 4 new spelling words each day. How many new spelling words do we have in a week?

15. Mary put 18 spools into 3 little boxes, putting the same number into each box. How many spools did she put into each box?

16. A lady had in her desk 15 envelopes. 6 of them were white, and the others yellow. How many yellow envelopes had she?

17. In one bookcase there were 9 books; in another 6 books; and in another 8 books. How many books were in the three bookcases?

18. A lady answered 9 telephone calls a day. In how many days did she answer 45 calls?

19. Add these columns:

7	8	9	8
8	7	2	5
6	5	6	3
2	3	4	1
+4	+6	+7	+6

LESSON SIX

COUNTING PINTS

I. NOTE TO TEACHER. Up to this point the problems solved by pupils have dealt with natural pluralities. That is, the units were naturally separated from one another, so that their number could be ascertained by simple counting. Now we come to deal with capacity, a property not naturally a plurality, but which may be thought of as such by the use of artificial units, the number of which can be ascertained by measuring or counting with the help of a standard of some kind.

In this exercise the artificial unit is the *pint;* and the standard or instrument which enables us to count these units is the *pint measure.*

Upon a table are placed a pint measure, a can filled with water, and an empty kettle, or small tub.

Teacher lifts the pint measure and says, "This is a pint measure. If I fill it with milk, I will have a pint of milk; if with water, I will have a pint of water. How many pints of water do you think are in this can?"

After various estimates, one of the pupils, Willie, is called to the table to count the pints in the can.

With no more help from the teacher than is necessary, Willie fills the pint measure from the can and empties it into the tub. He repeats this until all the water is transferred from the can to the tub, counting (aloud) the pints as he pours them out. Meantime the teacher is moving from the supply a counter for each pint. When Willie has finished, he says, "There were 9 pints of water in the can."

Teacher repeats, "Yes, there were 9." (Pointing to the 9 counters.)

THE FIRST ARTIFICIAL PLURALITY

KEY. 1. The quart measure. 2. The gallon measure. 3. The can. 4. The tub. 5. Willie. 6. The teacher. 7. A supply of counters. 8. The counters already taken from supply. 9. The last one being removed.

II. DRILL. Have pupils either use counters or describe the counting for each of the following:

1. There were 4 pints of milk in one can, 5 pints in another, 7 in another, and 9 in another. How many pints were in all the cans?

2. I have 15 pints of root beer, and wish to put it into 5 bottles, putting the same number of pints into each. How many pints will each bottle contain?

3. Each of 4 jugs held 8 pints of vinegar. How many pints of vinegar were in all the jugs?

4. There were 14 pints of milk in a can. How many pints remained after 6 pints were sold?

4. How many times can 4 pints of oil be taken from a can which holds 36 pints?

UNCLASSIFIED PROBLEMS

1. Drill every day on the following addition and subtraction combinations. They should be arranged differently each day.

4	5	4	8	7	2	4
+8	+9	+5	+9	+7	+7	+6
12	14	9	17	14	9	10
−4	−5	−4	−9	−7	−2	−4
12	14	9	17		9	10
−8	−9	−5	−8		−7	−6

2. There were 17 pints of oil in a can. After 8 pints had been used, how many pints remained?

3. A family used 24 pints of cider in 8 days, using the same number of pints each day. How much was that a day?

4. 7 of the children's Easter eggs were red; 9 were purple, and 5 were blue. How many Easter eggs had the children?

5. Two girls have together 14 buttons. One girl has 8 buttons. How many has the other?

6. A gallon measure will hold 8 pints of oil. It was filled 6 times, and each time emptied into a can. How many pints of oil were then in the can?

7. With the teacher's help, add these columns:

7	5	2	3
9	8	9	4
2	3	6	7
4	7	8	9
+6	+4	+7	+6

8. A little pear tree has 32 leaves on 4 branches. How many leaves are on each branch, if they have the same number apiece?

9. Edward writes 8 lines in his copybook each day. How many days will it take him to write 40 lines?

10. A little girl picked 3 bunches of violets. There were 8 violets in each bunch. How many violets did she pick?

11. In an orchard were 9 pear trees, 7 prune trees, and 6 apple trees. How many trees in the orchard?

12. 18 pencils were divided equally among 3 boys. How many pencils did each boy receive?

13. 3 street cars just passed our house. There were 12 passengers in each of them. How many passengers were in all the cars?

14. To how many customers can 4 pints of ice cream apiece be sold from a can containing 20 pints?

15. Florence made 4 rows of picture cards on the table. She put 9 cards in each row. How many picture cards did she use?

16. A lady used 32 eggs making cakes. She put 4 eggs into each cake. How many cakes did she make?

17. Mary had 10 tadpoles in a jar. One morning she saw that 4 tadpoles had turned into frogs. How many tadpoles were left?

18. 15 pints of milk filled 5 pitchers of the same size. How much did each pitcher hold?

19. On a pear tree were 15 pears. But the wind blew off 6. How many were left on the tree?

20. On his birthday a little boy received 63 pennies, 7 from each of his relatives. How many relatives gave him pennies?

21. A family bought 8 pints of cream one week, 7 the next, 9 the next, and 9 the fourth. How many pints did they buy in the four weeks?

22. A conductor rang up 22 fares on one trip and 26 on another. How many did he ring up on both trips?

23. There were 6 little girls at Helen's party. She gave each of them 5 pieces of candy. How many pieces of candy did it take?

24. Mary's mother bought 12 oranges and put them on a shelf in the pantry. 5 of them were used for dinner. How many were left on the pantry shelf?

25. A boy had 9 white marbles, 7 brown marbles, and 6 glass marbles. How many marbles did he have altogether?

LESSON SEVEN

COUNTING QUARTS

I NOTE TO TEACHER. Examine the exercise on page 37 entitled "Counting pints," and repeat it, using a quart measure instead of a pint. Incidentally the fact may be brought out that *2 pints = 1 quart*, but there is no occasion to memorize it, and it certainly should not be used as an unstated condition of problems. At present, for the reason stated on page 17, all the conditions of the problem should be clearly stated in the simplest language.

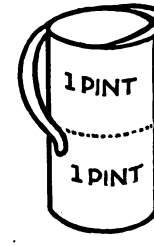
At a later stage the tables on pages 360 and 361 are to be learned, and the relations to one another of the various artificial units of a magnitude are to be used as implied conditions of problems. But the pupil is not ready for either of these steps at present.

II. DRILL. Have pupils either use counters or describe the counting in solving each of the following:

1. From 132 quarts how many times can 11 quarts be taken?

PINT

2. There were 7 quarts of water in one bucket, 6 in another, 4 in another, and 5 in a fourth. How many quarts of water were in all the buckets?



QUART

3. Each of 9 cans contained 5 quarts of molasses. How many quarts in all the cans?

4. A can holding 32 quarts of oil was emptied in 8 days. How much was used each day, if the same quantity was used each day?

UNCLASSIFIED PROBLEMS

1. Drill daily on the following:

4	5	3	6	2	5	6
+7	+8	+4	+6	+7	+7	+8
11	13	7	12	9	12	14
−4	−5	−3	−6	−2	−5	−6
11	13	7		9	12	14
−7	−8	−4		−7	−7	−8

2. In an orchard were 56 trees, planted 8 in a row. How many rows were there?

3. Add these columns:

6	8	2
7	4	6
2	7	8
3	5	4
+9	+3	+5

4. A milkman had a customer charged with 16 pints of cream. If the customer took 2 pints a day, how many days had the bill been running?

5. Two boys picked 15 quarts of cherries. But 6 quarts of them were unfit to use. How many quarts of good cherries did they pick?

6. Each pupil in a class read 3 stories. They all read 24 stories. No story was read twice. How many pupils were in the class?

7. 7 girls have together 63 doll dresses. How many has each girl, if all have the same number?

8. Harry read a story 72 pages long. If he read 9 pages each evening, how many evenings did it take to read the story?

9. William had 8 black tops, 7 red ones, and 5 white ones. He bought 3 more white ones and 4 more red ones. How many tops had he then?

10. If a family took 12 quarts of milk a week, how many quarts did they take in 9 weeks?

11. We have 45 rosebushes in 5 equal hedges. How many bushes in each hedge?

12. A keg contained 18 quarts of vinegar. After 9 quarts had been used, how much vinegar remained in the keg?

13. In two days a teamster fed 5 quarts of oats to each of his 4 horses. How many quarts did he feed to all of them?

14. In one box were 9 pencils, in another 7, in another 6, and in a fourth 8. How many pencils were in all the boxes?

15. A horse was fed 3 quarts of oats a day. How many days did 24 quarts last him?

16. From a 15-pint can of paint 7 pints were used. How many pints remained in the can?

17. In a bookcase are 6 shelves, with 9 books on each shelf. How many books are in the bookcase?

18. Each of 9 robins ate 7 cherries from George's tree. How many cherries did all the robins eat?

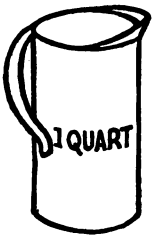
LESSON EIGHT

COUNTING GALLONS

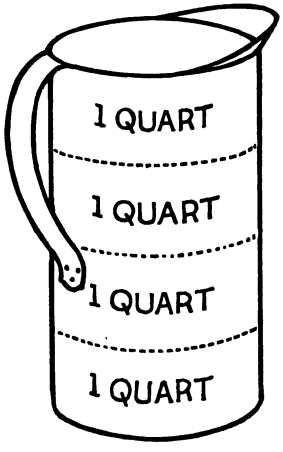
I. EXPLANATION. Proceed as in counting pints, page 37.

II. DRILL. Have pupils either use counters or imagine the counting in solving the following:

1. From a 12-gallon can of coal oil, 5 gallons were used. How many gallons remained in the can?
2. $\frac{1}{5}$ of 60 gallons = ?
3. How many gallons of vinegar are in 8 kegs, if 12 gallons are in each keg?
4. How many days would 12 gallons of coal oil last, if 3 gallons are used each day?



QUART



GALLON

UNCLASSIFIED PROBLEMS

1. Drill daily on the following:

3	4	7	5	9	2	3
+7	+8	+9	+6	+9	+6	+6
10	12	16	11	18	8	9
−3	−4	−7	−5	−9	−2	−3
10	12	16	11	14	8	9
−7	−8	−9	−6	−5	−6	−6

2. 12 quarts of molasses were poured into pitchers holding 2 quarts each. How many pitchers were used?

3. A barrel contained 14 gallons of vinegar. After 9 gallons were used, how many gallons remained in the barrel?

4. Bessie, Fay, and Mary each had 7 pieces of candy. How many pieces had they altogether?

5. 50 children went to a picnic in 5 wagons. Each wagon held the same number of children. How many were in each wagon?

6. In one tank there are 9 gallons of oil. In another tank there are 12 gallons. How many gallons in both tanks?

7. A painter used 32 panes of glass. He put 8 panes in each window. How many windows did he make?

8. From a can holding 15 pints of ice cream, 9 pints were used. How many pints of ice cream remained in the can?

9. Mary gathered 43 flowers, and Lily gathered 23 flowers. How many flowers did the two girls gather?

10. A druggist bought 4 kegs of lime water, each containing 8 gallons. How many gallons of lime water did he buy?

11. A lady had 14 jars of fruit in her pantry. She used 2 jars each week. How many weeks did the fruit last?

12. John picked 7 quarts of cherries. Ann picked 8 quarts. Tom picked 9 quarts. How many quarts of cherries did they all pick?

13. 6 pitchers each holding 3 pints of water were poured into an empty bucket. How many pints of water were then in the bucket?

14. A lady planted 15 tomato plants. 6 of them died. How many grew?

15. How many 5-gallon cans can be filled from 55 gallons of oil?

16. In a snowball fight, there were 9 boys on one side, 8 boys on the other, and 6 boys looking on. How many boys were present?

17. There are 17 boys in a class. Yesterday 8 of them were absent. How many were present?

18. On each of 9 plates there were 2 oranges. How many oranges were on all the plates?

LESSON NINE

READING AND WRITING NUMBERS TO 600

I. EXPLANATION. If pupils can read and write numbers well to 400, there is nothing new in this exercise, except the use of the figures 5 and 6, in the hundreds' place.

The numbers 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 500 to 509, must be frequently read and written, attention being called to the use of the naughts, namely, to put the 4 or 5 in the third, or hundreds' place.

Exercise. "What does a figure in the first place represent?" *Answer.* "Units or ones."

Show four counters, and write 4.

"What does a figure in the second place represent?" *Answer.* "Tens."

Show four columns of ten counters, and write 4.

"Is this 4 in the second place?" "No; in the first."

"How will I put it in the second?"

"Place a naught to the right, thus 40."

"What does a figure in the third place represent?" *Answer.* "Hundreds."

Show four hundreds (each hundred is a group of ten columns of ten counters each), and write 4.

"Is this 4 in the third place?"

"No; in the first."

"How will I put it in the third place?"
"Place two naughts to the right, thus, 400."

Exercise.

"Write four hundred one."
"In what place must the 1 be?"
"In the first place."
"Very well, write it there, thus 1."
"In what place must the 4 be?"
"In the third place."
"How can we put it there?"
"Write it to the left of the 1 with a naught between them, thus 401."

II. DRILL. Write the following:

1. Four hundred one.
2. Four hundred ten.
3. Five hundred two.
4. Five hundred twenty.
5. Four hundred three.
6. Four hundred thirty.
7. Four hundred four.
8. Four hundred forty.
9. Five hundred five.
10. Five hundred fifty.
11. Two hundred six.
12. Two hundred sixty.

Read the following numbers:

1. 10
2. 200
3. 360
4. 501
5. 524
6. 600
7. 406
8. 508
9. 470
10. 409

"What is the use of the naught in (1)?"

Answer. "The 1 expresses tens, and must be in the second place. The naught is necessary to put it there."

Let all the naughts be accounted for in this way.

In (5) tell why no naughts are needed.

UNCLASSIFIED PROBLEMS

1. Drill every day on the following:

3	3	5	8	5	2	4
+5	+8	+9	+9	+5	+5	+6
8	11	14	17	10	7	10
−3	−3	−5	−8	−5	−2	−4
8	11	14	17		7	10
−5	−8	−9	−9		−5	−6

2. Write: five hundred six; five hundred sixty; five hundred sixty-three; four hundred eight; four hundred eighteen; four hundred eighty; four hundred eighty-one.

3. Mary had five dolls. One of them had 6 dresses, another 9 dresses, another 8 dresses, another 7 dresses, and the fifth one 5 dresses. How many dresses had the five dolls?

4. Mary has 30 doll dresses for her 5 dolls. How many is that apiece, if all have the same number?

5. A cow gave 18 pints of milk. 2 pints were sold to each customer. How many customers were supplied, when the milk was all sold?

6. A boy had 5 bags with 9 marbles in each. How many marbles had he in all the bags?

7. On Christmas night Mary had 9 candles at each window of their front room. There were 4 windows in the room. How many candles did she use?

8. A tailor put 8 buttons on each overcoat he made. He used 24 buttons. How many overcoats did he make?

9. A lady preserved 9 pints of blackberries, 7 pints of strawberries, and 8 pints of huckleberries. How many pints of berries did she preserve?

10. There were 15 children in our arithmetic class, but 6 of them left school. How many are now in the class?

11. A barrel contained 17 gallons of vinegar. After 9 gallons were used, how many gallons remained?

12. 7 buckets of water were poured into an empty tub. Each bucket held 5 gallons. How many gallons of water were then in the tub?

13. 36 pupils in a class marched 4 in each rank. How many ranks were there?

14. There are 8 tables in an ice cream parlor. 4 persons are eating ice cream at each table. How many persons are eating ice cream in the parlor?

LESSON TEN

(a) ADDING LARGE NUMBERS, USING EXPONENTS

I. EXPLANATION. Suppose a concrete problem which gives this numerical expression: 45 + 29 + 37 + 84 + 52.

Beginning with the right or units column, we find its sum to be 27. Write the 7 as in adding a single column, but instead of writing the 2 to the left of the 7, write it below the 5 as shown in the figure. Commencing with the 2, we add the second or tens column, and find its sum to be 24 tens. Write the 4 tens under the tens column, and the 2 hundreds to the left of the 4. The sum, then, is 247.

$$\begin{array}{r} 4\ 5 \\ {}^{0}\ {}^{2} \\ 2\ 9 \\ \quad\ {}^{3} \\ 3\ 7 \\ \quad\ {}^{5} \\ 8\ 4 \\ +5\ 2 \\ \quad\ {}^{2} \\ \hline 247 \end{array}$$

II. DRILL. Find the sums of the following:

(1)	(2)	(3)	(4)	(5)
145	79	59	48	29
97	64	178	67	64
86	57	95	94	37
34	39	136	38	29
+167	+46	+84	+24	+46

(b) SUBTRACTING LARGE NUMBERS WITHOUT CARRYING

I. EXPLANATION. *Problem.* From a flock of 645 sheep 214 were sold. How many were kept? Imagined counting gives the expression,

$$\begin{array}{r} 645 \\ -214 \\ \hline \end{array}$$

Notice that each figure of the number to be taken away (214) is less than the figure above, and can be taken from it. Performing this operation we have,

$$\begin{array}{r} 645 \\ -214 \\ \hline 431 \end{array}$$

That is, 431 sheep were kept.

UNCLASSIFIED PROBLEMS

1. Drill daily on the following combinations:

2	4	6	4	6	4	3	3
+7	+4	+7	+6	+8	+9	+5	+6
9	8	13	10	14	13	8	9
−2	−4	−6	−4	−6	−4	−3	−3
9		13	10	14	13	8	9
−7		−7	−6	−8	−9	−5	−6

2. Write: five hundred nine; five hundred ninety; five hundred nineteen; four hundred eight; four hun-

dred eighty; four hundred eighteen; three hundred seven; three hundred seventy; three hundred seventeen; two hundred six; two hundred sixty; two hundred sixteen; one hundred five; one hundred fifty; one hundred fifteen.

3. A milkman sold 22 quarts of milk on one street; 15 quarts on another; and 17 quarts on another. How many quarts did he sell on the three streets?

4. Each of the 9 pupils in our class spelled 6 words. How many words did they all spell?

5. Lucy found 20 eggs in 5 nests. How many did she find in each, if all had the same number?

6. One barrel contained 27 gallons of rain water; another contained 24 gallons; a third 29 gallons; and a fourth 31 gallons. How many gallons of rain water did the four barrels hold?

7. There were 325 gallons of water in a tank. 113 gallons were used. How many gallons were left in the tank?

8. A baker made 34 lemon pies, 29 peach pies, and 28 prune pies. How many pies did he make?

9. 27 boys were divided into 3 ball teams. How many boys were in each team?

10. A boy had 38 rabbits; 21 of them ran away. How many rabbits had the boy left?

11. A boy gathered 26 eggs one day; 22 the second day; 19 the third day, and 24 the fourth day. How many eggs did he gather on all four days?

12. 2 pints of ink were placed in each of 7 bottles. How much ink was in all the bottles?

13. Samuel picked 28 plums from 4 trees. He picked the same number from each tree. How many plums did he get from each tree?

14.
$$\begin{array}{r} 183 \\ 184 \\ 79 \\ +46 \\ \hline \end{array}$$

15. Each of 6 thick bottles contained 8 pints of acid. How much acid in all the bottles?

16. A boy bought 3 neckties. He paid 25 cents for one, 16 cents for another, 29 cents for another. How many cents did he pay for the three?

17. How many balls are there in 3 Roman candles, each containing 8 balls?

18. John paid 36 cents for a reader and 8 cents for a slate. How many cents did he pay for both?

19. If 27 days since the term began have been sunny and 17 cloudy, how many days have passed?

20. My bookcase contains 21 books on the top shelf, 17 on the next, 19 on the next, 15 on the next, and 14 on the bottom shelf. How many books are there in the bookcase?

21. A milkman had four cans of milk on his wagon. containing 24 quarts, 22 quarts, 20 quarts, and 19 quarts, respectively. How many quarts of milk were in the four cans?

LESSON ELEVEN

COMPARISON OF NUMBERS

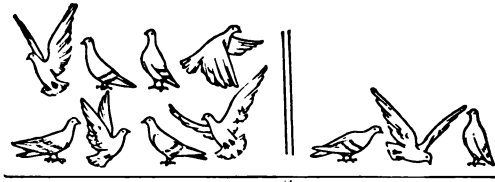
I. EXPLANATION. *Problem.* John has 4 pigeons, and William has 3 more pigeons than John. How many pigeons has William?

SOLUTION BY COUNTING

Take John's pigeons from the supply.
Take the same number of William's pigeons.
Take 3 more for William.
Count William's.
Arrange thus:

John's pigeons,

William's pigeons,



Part of W's = to J's.	Part of W's more than J's.

It will be observed that 11 counters are employed in solving the problem although the sum of the two parts is only 7. This is because one of the two parts is not stated directly, but by comparison with another unity which does not enter into the problem.

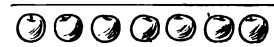
The numerical expression is $\underline{+3}$, in which both numbers represent parts of William's pigeons.

Second Problem. Mary has 12 apples, and Elizabeth has 7. Mary has how many more apples than Elizabeth?

Take Mary's apples from the supply and place them in a row.

Take Elizabeth's apples from the supply and place them in a row beside Mary's.

Thus:

	Part of Mary's = to Elizabeth's.	Part of Mary's more than Elizabeth's
Mary's apples,		
Elizabeth's apples,		

Notice that the function of Elizabeth's apples is to separate Mary's into two parts, 7 and 5.

The numerical expression is $\begin{array}{r} 12 \\ -7, \\ \hline 5 \end{array}$ in which both the 7 and the 5 stand for parts of *Mary's* apples.

II. DRILL.

1. John has 9 tops, which is 2 more than Samuel has. How many has Samuel?

2. Wilson has 16 books, which is 13 less than Thomas has. How many has Thomas?

3. Harry has 5 pictures, Mary has 9. How many more has Mary than Harry?

4. How many pictures less than Mary has Harry?

UNCLASSIFIED PROBLEMS

1. Drill on the following combinations:

6	3	3	4	5	8	3	4
+9	+9	+4	+7	+8	+9	+8	+8
15	12	7	11	13	17	11	12
−6	−3	−3	−4	−5	−8	−3	−4
15	12	7	11	13	17	11	12
−9	−9	−4	−7	−8	−9	−8	−8

2. Write the following: one hundred nine; one hundred ninety; two hundred three; two hundred thirty; three hundred four; three hundred forty; four hundred five; four hundred fifty; five hundred seventy-five; two hundred thirty-four; etc.

Justify the naughts used.

3. 7 boys have 63 chestnuts. If they have the same number apiece, how many chestnuts has each boy?

4. Each of 4 tin buckets held 3 quarts of cider. How much cider was there in all the buckets?

5. A fruit grower used 124 gallons of spray on his apple trees, 84 gallons on his pear trees, and 56 gallons on his peach trees. How many gallons of spray did he use?

6. A farmer owns 27 cows. His neighbor has 18 more. How many cows has his neighbor?

7. Henry has 8 times as many pigeons as Samuel. Samuel has 4 pigeons. How many has Henry?

8. Mary had 12 photographs taken. Charles had 4 less than Mary. How many photographs did Charles have taken?

9. There were 17 chickens in one yard, and 14 in another. How many chickens in both yards?

10. A milkman started out with 98 pint bottles of cream. He served 84 of them to his customers. How many pints of cream did he bring back?

11. When Martha was 8 years old, her father bought her 48 candles, so that she might place 8 candles in each of her birthday cakes. How many cakes had she?

12. Matthew left home with 53 marbles in his bag. When he returned he had only 31 marbles. How many did he lose?

13. Mr. Smith's water tank holds 165 gallons. Mr. Jones's tank holds 78 gallons more than Mr. Smith's. How much water does Mr. Jones's tank hold?

14. John's father worked in a box factory. In 3 days he made 27 boxes. How many boxes was that a day, if he made the same number every day?

15. There were 243 roses on one hedge, and 132 on another. How many more roses were on the first hedge than on the second?

16. In a schoolroom were 32 pupils. They recited in groups of 8. How many groups were there?

17. Mr. Johnson has 84 more sheep than Mr. Whitney. Mr. Whitney has 183 sheep. How many has Mr. Johnson?

18. Mrs. Waters set 6 hens, putting 12 eggs under each. How many eggs did she use?

19. 18 birds were put into cages, 3 into each cage. How many cages were used?

20. Mr. James had 28 horses, and sold $\frac{1}{4}$ of them. How many horses did he sell?

21. There were 358 sailors on one ship, and 134 less on another. How many sailors were on the second ship?

22. Each of 6 boys had 5 books. How many had they all?

23. William has paid 10 cents for a ball, and 36 cents for a book. How much money did he spend for both?

24. How much do I pay for a reader at 22 cents, a slate at 8 cents, and a copy book at 6 cents?

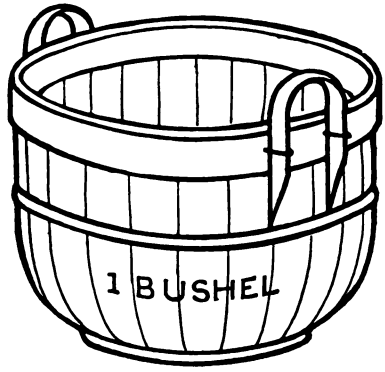
25. A newsboy sold 19 morning papers and 7 evening papers. How many did he sell in all?

26. After school on Monday James sold 32 papers, and during the next five days he sold 27, 19, 31, 41, and 23. How many did he sell during the week?

27. The oatmeal for breakfast cost 12 cents, the milk 6 cents, the fruit 10 cents, the rolls and butter 8 cents, and the eggs 16 cents. How much did this food cost?

LESSON TWELVE

COUNTING PECKS AND BUSHELS



I. EXPLANATION. (a) Show the pupils a *peck* measure, and describe an exercise similar to the counting of pints. The presence of the measure will stimulate the imagination so that the process will be understood without actually measuring anything.

(b) Show the pupils a bushel measure and with it conduct an exercise similar to (a).

These units may now be introduced in concrete problems without further drill.

It is to be remembered that the main purpose of these exercises is to help the pupil to use counters to represent artificial units. At the same time the presence of the measures gives him a clear notion of the significance of the terms used to name these units.

REVIEW DRILLS

1. Drill on the following:

3	5	3	7	9	2	2
+7	+7	+9	+8	+9	+9	+5
10	12	12	15	18	11	7
−3	−5	−3	−7	−9	−2	−2
10	12	12	15		11	7
−7	−7	−9	−8		−9	−5

2. Write the following numbers: five hundred one; five hundred ten; four hundred two; four hundred twenty; four hundred twelve; three hundred twenty-seven; three hundred seventy-two; etc.

Justify the naughts.

UNCLASSIFIED PROBLEMS

1. 9 pecks of potatoes were sold to each family. How many families would 72 pecks supply?

2. In one field 46 bushels of wheat were raised, and in another field 15 more bushels were raised than in the first field. How many bushels of wheat were raised in the second field?

3. How many pecks of apples in 8 barrels, if each barrel holds 10 pecks?

4. A farmer used each week 6 bushels of bran for cow feed. In how many weeks did he use 42 bushels?

5. There were 97 bushels of corn in a crib. 46 bushels were fed to the horses. How many bushels remained in the crib?

6. There were four rafts of logs in the river. The first had 45 logs in it; the second 63 logs; the third 29 logs; and the fourth 78 logs. How many logs in all four rafts?

7. There were 3 water coolers in a waiting room, with 6 gallons of water in them. If there was the same number of gallons in each cooler, how much water was in each?

8. 40 pecks of potatoes were put into sacks, 5 pecks in each sack. How many sacks were used?

9. In an ice cream parlor, 24 people were seated around 4 tables, the same number at each table. How many were at each table?

10. A grocer delivered 2 gallons of vinegar to each of 7 families. How many gallons did he deliver to them all?

11. It took 9 jugs of the same size to hold 18 gallons of cider. How much cider was in each jug?

12. A farmer raised 792 bushels of oats. If he kept 371 bushels for his own horses, how much did he sell?

13. In a Sunday school there were 15 pupils in one class, 19 in another, 22 in a third, and 27 in a fourth. How many pupils were in the four classes?

14. 2 little girls each paid 9 pins to go to a doll show. How many pins did they both pay?

LESSON THIRTEEN

COUNTING OUNCES AND POUNDS

I. EXPLANATION. Place on the table a weighing balance, ten or twelve ounce weights, a bucket of sand, with a scoop, and several paper bags.

Teacher lifts an ounce weight and says, "How heavy do you think this is?"

After various guesses she continues, "It weighs an ounce. All of these (pointing to the ounce weights), weigh an ounce apiece."

She then fills a small paper bag with sand and continues. "How heavy do you think this bag of sand is?"

After various guesses, she says, "Let us see." She then places the bag on one pan of the balance, and on the other pan places ounce weights, one at a time, until a balance is obtained, counting the weights as they are placed in the pan.

IT WEIGHS SEVEN OUNCES

"It weighs 7 ounces."

Another bag is filled, and is found to weigh 8 ounces. Still another holds 10 ounces.

"How many ounces in the three bags?"

Probably the pupils will find this at once by computation. If not, the counters may be employed.

The pound weight may now be shown the pupils, and its relation to the ounce found with the balance. This, however, is merely incidental, and at this stage is not to be memorized or used in problems.

II. DRILL. 1. From a bag holding 28 ounces of sugar 8 ounces were used to make a cake. How many ounces remained in the bag?

2. 3 large paper bags contained 9 pounds of sugar apiece. How much sugar was in all the bags?

3. 12 ounces of candy were divided equally among 4 children. How many ounces did each child receive?

REVIEW DRILLS

1. Drill daily on these combinations:

4	7	6	5	3	7	4
+9	+8	+6	+9	+5	+7	+6
13	15	12	14	8	14	10
−4	−7	−6	−5	−3	−7	−6
13	15		14	8		10
−9	−8		−9	−5		−4

2. Write the following: five hundred ninety-seven; four hundred eight; three hundred ninety; two hundred seven; one hundred one, etc.

Justify the naughts.

UNCLASSIFIED PROBLEMS

1. An apple grower packed his apples in barrels that held 2 bushels each. How many barrels would be needed for 10 bushels?

2. In the cellar there were 49 pecks of white potatoes, and 37 pecks of sweet potatoes. How many pecks of potatoes were there?

3. A lady put 36 ounces of candy in 4 boxes, so that they all had the same number of ounces. How many ounces of candy were in each box?

4. Our tea can holds 12 ounces of tea. Yesterday mother put 8 ounces into the can, and just filled it. How many ounces were already in the can?

5. In a cellar were 54 gallons of cider in 6-gallon kegs. How many kegs were there?

6. A boy picked 6 baskets of cherries. Each basket held 8 pounds. How many pounds of cherries did he pick?

7. A grocer sold to one family 9 ounces of tea; to another 8 ounces; and to a third 14 ounces. How many ounces of tea did the three families buy?

8. George picked 9 quarts of berries. Richard picked 3 quarts less than George. How many quarts did Richard pick?

9. There were 9 pounds of butter in each of 8 butter tubs. How many pounds of butter altogether?

10. A farmer sold 118 sheep. If he had 249 at first, how many were left?

11. 27 pounds of butter were made into 9 equal rolls. How many pounds were put into each roll?

12. If 8 ounces of silver are used in making a dish, how many such dishes will 32 ounces make?

13. One boy weighs 64 pounds, and another boy weighs 19 pounds more than the first. What is the weight of the second boy?

14. Mary had 124 needles. Josephine had 13 needles less than Mary. How many needles had Josephine?

15. The mason brought 4 baskets of lime to make mortar. In each basket were 5 pecks. How many pecks of lime did he bring?

16. A lady used 56 ounces of flour to bake 8 loaves of bread of the same weight. How many ounces of flour were in each loaf?

17. John weighs 56 pounds; William 63 pounds; and Henry 97 pounds. How many pounds do the three boys weigh?

18. When Henry went to visit his uncle, he had to ride 38 miles in an automobile. When he had ridden 22 miles, how far had he yet to ride?

19. How far had Henry ridden when he had only 8 miles farther to ride?

LESSON FOURTEEN

COUNTING YEARS AND MONTHS

I. EXPLANATION. Teacher says, "Martha, how old are you?"

"Seven."

Teacher places 7 counters on the table.

"In 3 more years (placing 3 more counters on the table), how old will Martha be?"

Hands are shown, and the one called on answers, "Ten."

"Martha, you may write the story on the board."

Martha writes,

$$\begin{array}{r} 7 \\ +3 \\ \hline 10 \end{array}$$

"A boy began to go to school when he was 6 years old, and left when he was 14 years old. How many years did he go to school?"

"How old was he when he left?"

"Fourteen."

"Then take 14 counters from the supply."

"How many of these years did he live before he went to school?"

"Six."

"Take 6 of them away from the 14."

"Now how many are left in which he went to school?"

"Eight."

"You may all write the story."

Pupils write,

$$\begin{array}{r} 14 \\ -6 \\ \hline 8 \end{array}$$

"How many months in the year, class?"

If no one knows, teacher answers, "Twelve." The twelve months are named, and their names written on the blackboard, thus:

January	April	July	October
February	May	August	November
March	June	September	December

A counter is moved from the supply as each month is named.

"How many months do you have vacation?"

"Two."

Two of the counters are moved from the 12.

"Then how many months are left for school?"

"Ten."

"Write the story."

Pupils write,

$$\begin{array}{r} 12 \\ -2 \\ \hline 10 \end{array}$$

A lady made 3 trips to Europe. Each trip she was gone 4 months. How long was she gone altogether?

Pupils write, $3 \times 4 = 12$.

REVIEW DRILLS

1. Drill daily on these combinations:

7	5	2	6	4	3	5
+9	+6	+5	+9	+5	+6	+7
16	11	7	15	9	9	12
−7	−5	−2	−6	−4	−3	−5
16	11	7	15	9	9	12
−9	−6	−5	−9	−5	−6	−7

2. Dictate numbers to be written not higher than 600. Often dictate those which contain naughts. Emphasize the function of the naughts.

UNCLASSIFIED PROBLEMS

1. A pupil went to school 27 months in 3 years. He attended the same length of time each year. How many months a year did he go to school?

2. A hunter bought 6 pounds of meat for each of his 4 dogs. How much meat did he buy?

3. Mr. St. Clair was in Europe 27 months. This was 17 months longer than Mr. Williams was in Europe. How long was Mr. Williams in Europe?

4. At a party, 9 children were given 4 ounces of candy each. How much candy was used?

5. A boy worked in several stores, 6 months in each. In 24 months, at how many places had he worked?

6. 27 sandwiches were put into boxes, 3 sandwiches in each box. How many boxes were used?

7. John was born in Salem, and lived there 6 years. He then moved to Oregon City, and lived there 5 years. He next moved to Portland where he has lived 4 years. How old is John?

8. 18 pounds of spice filled 3 boxes of the same size. How many pounds were in each box?

9. 8 boxes contained 9 children's tops apiece. How many tops were in all of them?

10. 45 pounds of coffee were put up in 5-pound packages. How many packages were there?

11. There were 34 pigeons in one pigeon house. In a second pigeon house there were 15 more pigeons than in the first. How many pigeons were in the second house?

12. Samuel had 39 cherries, and ate 27 of them. How many had he left?

13. A man who is 24 years old, has spent his life in 3 cities. He has lived the same time in each. How many years has he spent in each city?

14. A logger has 42 oxen, which he uses to drag logs to the creek. If there are 6 oxen in a team, how many teams has he?

15. How many quarts of cider in 5 jugs, if each jug holds 3 quarts?

16. A gentleman was absent 25 months. He spent 12 months in New York, and the rest of the time in Boston. How long was he in Boston?

17. A boy picked 9 quarts of berries on Monday, 11 quarts on Tuesday, 12 quarts on Wednesday, and 8 quarts on Thursday. How many quarts did he pick in the four days?

18. A boy gathered 32 eggs on Monday, 25 on Tuesday, 29 on Wednesday, 36 on Thursday, and 24 on Friday. How many did he gather in the five days?

19. There are 54 children to-day in the primary grades of a school. This is 13 more than were present yesterday. How many primary pupils were in school yesterday?

20. I read 28 pages of my book yesterday, and 14 pages this morning; I have 90 pages yet to read. How many pages are there in my book?

21. The A class had 59 pupils, and the B class 47. How many more were there in the A class than in the B class?

22. If Lucy lives 8 blocks from the schoolhouse and can walk a block in 2 minutes, in how many minutes can she walk to school?

23. In the library of the Boys' Club there are 20 books of travel, 6 books of poems, and 26 story books. How many books are in the library?

24. 8 boys have 9 nails apiece. How many nails have they all?

LESSON FIFTEEN

COUNTING DAYS AND WEEKS

I. EXPLANATION. "How many days in a week?"

"Seven."

Class names them, and teacher writes the names on the board, thus:

Sunday		
Monday✓	Wednesday✓	Friday✓
Tuesday✓	Thursday✓	Saturday

"On which days do you go to school?"

As the school days are named, teacher checks them as shown above.

"On which days is there no school?"

Class answers.

"Again. How many days in the week?"

"Seven."

"On how many days is there school?"

"Five."

"On how many days is there no school?"

"Two."

"You may write the story."

Pupils write,

$$\begin{array}{r} 7 \\ -5 \\ \hline 2 \end{array}$$

II. DRILL. Describe the counting in each of the following problems:

1. Mary spent 24 weeks visiting 3 eastern cities. If she divided her time equally among them, how long did she visit each?

2. If it takes Louise 3 days to read a book, how many such books can she read in 21 days?

UNCLASSIFIED PROBLEMS

1. Drill on addition combinations with corresponding subtractions.

2. Dictate numbers to be written, then read. Emphasize function of naughts.

3. Last year we planted 17 rosebushes, but 8 of them have not yet bloomed. How many have bloomed?

4. A boy attended 4 schools for 12 years. He spent the same time in each school. How many years was he in each?

5. Martin left home with his father and traveled 13 weeks. They then spent 9 weeks on a farm. Martin then went to a boarding school for 24 weeks. After that he went home again. How many weeks had he been away?

6. Mr. Brewster's cows each gave 5 quarts at a milking. He got, altogether, 40 quarts at a milking. How many cows had he?

7. There are 96 days in the present school term, and 45 days are gone. How much of the term remains?

8. 42 pounds of sugar were put into sacks, 6 pounds in each sack. How many sacks were used?

9. 35 pints of root beer were put into jugs holding 5 pints each. How many jugs were used?

10. On a Christmas tree were 8 sacks of candy. If there were 3 ounces in each sack, how many ounces of candy were there on the tree?

11. A little girl had 14 ounces of candy. She gave some of it to her friends and had left 5 ounces. How much did she give away?

12. A party of boys camped in the woods 18 days. During that time they were in 3 camps, spending the same time in each. How many days were they in each camp?

13. John went to the country to visit 3 cousins. He spent 7 days with each. How long did he stay in the country?

14. How much vinegar will it take to fill 6 jugs, if each jug holds 4 quarts?

15. The policeman in our neighborhood is changed every 3 months. In 12 months how many different policemen have we had?

16. A hotel has 18 rooms on the first floor, 21 on the second, 24 on the third, and 24 on the fourth. How many rooms in the hotel?

17. A grocer had some small butter tubs which held 9 pounds each. How many were needed to hold 36 pounds of butter?

18. A butcher took a piece of meat weighing 20 pounds and divided it equally among 5 families. How many pounds did each family receive?

CHAPTER II

STEPS OF THIRD YEAR

LESSON SIXTEEN

COUNTING HOURS AND MINUTES

I. EXPLANATION.

"How many hours in a day?"

After various guesses, teacher says, "twenty-four." (Counting 24 counters from the supply.)

"How many hours do you sleep each day?"

After various statements, teacher says, "about nine." (Taking 9 counters from the 24.)

"Then, how many hours are you awake each day?"

"Fifteen."

"You may write the story."

Pupils write,

$$\begin{array}{r} 24 \\ -9 \\ \hline 15 \end{array}$$

"A boy went to school 3 hours; played 4 hours, and worked 2 hours. How many hours did he use?"

If the method of computation is not at once apparent to the class, use counters.

"A street car makes a trip in 12 minutes. How long will it take to make 3 trips?"

If the computation is not obvious, get it from counting.

REVIEW DRILLS

1. Drill on the addition table and the corresponding subtractions.

2. Dictate numbers as large as 700. Let many of them contain naughts. Point out the office of the naughts.

UNCLASSIFIED PROBLEMS

1. Each pupil in a class reads 2 minutes. How many pupils can read in 24 minutes?

2. In a jeweler's window were 29 rings with diamonds in them; 36 with rubies; 42 with pearls, and 62 without stones. How many rings were in the window?

3. 7 boys had 9 tops each. How many tops had they altogether?

4. 48 hours is a full week's work. In one week a workman was idle 17 hours. How many hours did he work?

5. 7 bushels of oats were fed each day to a herd of horses. How many bushels were used in 9 days?

6. In a week 5 dogs were fed 35 pounds of meat. How many pounds apiece was that, if all received the same quantity?

7. A bricklayer worked 6 days each week for 8 weeks. How many days did he work altogether?

8. One month our class solved 76 problems in arithmetic. Another month it solved 83 problems. During a third month it solved 64 problems. How many problems did it solve during the three months?

9. A newsboy started out with 196 papers, and returned with 24. How many did he sell?

10. A girl had 24 buttons sewed on 3 cards. If there was the same number on each, how many buttons were on each card?

11. Ella practices her music 2 hours a day. In how many days does she practice 14 hours?

12. There were 548 pupils in our school. When measles broke out, 226 pupils left school. How many remained?

13. After dinner Esther played with her kitten 15 minutes; talked with her mother 15 minutes; studied her lessons 35 minutes; read a book 25 minutes, and then went to bed. How many minutes did Esther remain up after dinner?

14. Mary had 27 minutes to write letters. It took 9 minutes for each letter. How many did she write?

15. 24 pints of paste were bought in bottles holding 4 pints each. How many bottles were bought?

16. Our school year is 30 weeks long, and is divided into 3 equal terms. How many weeks in each term?

17. 8 jugs of cider held 4 quarts apiece. How many quarts were in the 8 jugs?

18. A teacher had 80 penholders in a box. She gave out 30 to her pupils. How many remained in the box?

19. One sack of salt contained 15 quarts; another sack held 21 quarts; another 19 quarts, and another 12 quarts. How many quarts of salt in the four sacks?

20. A ferryboat made 8 trips in 72 minutes. How many minutes was that for each trip, if all the trips took the same number of minutes?

21. Our reader contains 194 pages. We have read 53 pages. How many pages have we yet to read?

22. George had 37 firecrackers. Henry had 17 more than George. How many firecrackers had Henry?

23. A farmer had 48 cows in 4 stables. The same number of cows were in all the stables. How many cows were in each?

24. It takes 8 quarts of oil to fill a can. How many quarts does it take to fill 9 cans of the same size?

25. There are 7 days in a week. How many weeks are in 42 days?

26. In a schoolroom are six rows of desks. 7 pupils are seated in the first row; 8 in the second, 6 in the third, 7 in the fourth, 6 in the fifth, and 5 in the sixth. How many pupils are in the room?

LESSON SEVENTEEN

TRANSFORMATION OF FUNCTION

I. EXPLANATION. *Problem.* A farmer sold 6 cows, and had 9 remaining. How many had he at first?

Teacher going to the number table, takes from the supply an uncounted group of counters (about 20).

"Suppose these are the cows the farmer had at first. How many did he sell?"

"Six."

Teacher counts out 6 from the uncounted group.

"And how many were left?"

"Nine."

Teacher counts nine of the uncounted group and returns the remainder to the supply saying,

"We took out too many."

"How many had he at first?"

Class counts and answers "fifteen."

"You may write the story."

Class writes,

$$\begin{array}{r} 6 \\ +9 \\ \hline 15 \end{array}$$

Problem.

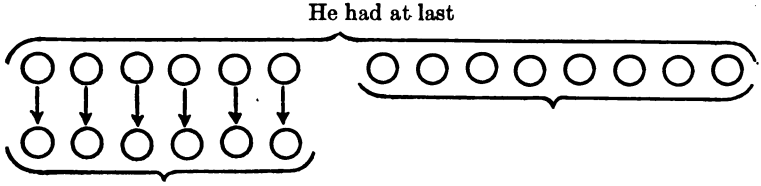
A boy bought 6 marbles and then had 14. How many had he at first?

"How many marbles had the boy at last?"

"Fourteen."
Teacher takes 14 from the supply.
"How many of these did he buy?"
"Six."
Teacher counts 6 from the 14.
"How many then had he at first?"
Pupils count and answer, "eight."
"You may write the story."

Class writes,

$$\begin{array}{r} 14 \\ -6 \\ \hline 8 \end{array}$$



NOTE TO TEACHER. It will be noticed that 6 marbles in the objective solution are taken from the 14. In the real situation, however, they were added to the 8 marbles which the boy had at first, i.e., a real addend is changed in computation to a subtrahend. Its function is transformed.

REVIEW DRILLS

1. Drill daily on addition table and corresponding subtractions.

2. Dictate numbers as high as 700. Afterwards have them read. Let numbers containing naughts

often occur. When these are read have the office of the naughts pointed out.

UNCLASSIFIED PROBLEMS

1. A boy had 275 marbles. He traded 143 of them for tops. How many marbles had he left?

2. A man worked 42 hours one week, 37 the next, and 38 the next. How many hours did he work in the three weeks?

3. Emily practices 3 hours every day but Sunday. How many hours does she practice in a week?

4. Martin bought 7 rabbits. He then had 9. How many rabbits had he before buying the 7?

5. A lady wrote 4 stories in 20 days. If she spent the same time on each story, how many days did it take to write each?

6. There were 54 ounces of powder in one can, and 23 ounces less in another. How many ounces of powder were in the smaller can?

7. A teacher works 6 hours every day. How many hours does she work in 5 days?

8. After a farmer had sold 47 pecks of potatoes, he had left 35 pecks. How many pecks had he at first?

9. John Emmons was in the navy 24 years. He served 3 years each term. How many terms did he serve?

10. A storekeeper bought 23 pounds of candy. He then had 97 pounds. How much had he at first?

11. Every summer Henry visits his grandmother in the country, staying 6 weeks. How many visits must he make to spend 54 weeks with his grandmother?

12. A man bought furniture, paying installments on it every 8 weeks. How many installments did he pay in 56 weeks?

13. From 976 take 824.

14. After selling 29 heads of cabbage, a gardener had 48 heads left. How many heads were in his garden at first?

15. Add,

$$\begin{array}{r} 564 \\ 208 \\ 517 \\ 634 \\ +297 \\ \hline \end{array}$$

16. 9 girls had 6 books apiece. How many books had they all?

17. Add,

$$\begin{array}{r} 275 \\ 368 \\ 456 \\ 584 \\ +236 \\ \hline \end{array}$$

18. 7 cakes of soap of equal weight weighed 21 ounces. What did each cake weigh?

LESSON EIGHTEEN

COUNTING INCHES, FEET, AND YARDS

I. EXPLANATION. *Linear Magnitude.* Have upon the table a foot rule, a yardstick, and some pieces of cardboard, each an inch long.

Teacher says, "How long is this piece of cardboard?"

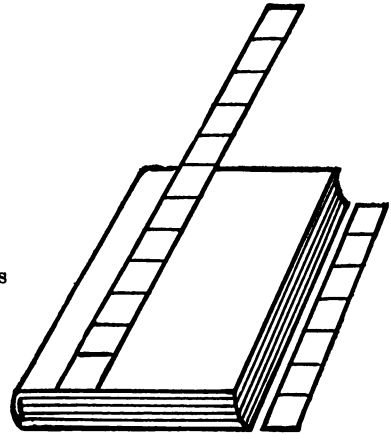
After various guesses she says, "It is an inch long."

"How long is this rule?" (Showing the foot rule.)

After guesses teacher says, "We will measure it and see."

Twelve pieces of cardboard are placed end to end upon the rule, and are found to coincide with it in length.

THE FOOT RULE



"It is 12 inches long."

"We also say it is a foot long."

"How long is this book?"

The book is measured, at first with the pieces of cardboard, afterwards with the rule, and found to be 7 inches long.

"How wide is it?"

Its width is measured with the rule alone, and found to be 4 inches.

Teacher says,

"I have a piece of ribbon 8 inches long, another 5 inches long, and another 13 inches long. How long are they all?"

If the method of computation is not obvious, teacher draws three lines on the blackboard, each representing a piece of ribbon.

"How many inch pieces must I put along this to measure it?" (Pointing to the line marked 8.)

"Eight."

Teacher takes 8 one-inch pieces from the supply and puts them in a row on the number table.

"How many to measure this?" (Pointing to the line marked 5.)

"Five."

Teacher takes 5 more one-inch pieces from the supply, and with them prolongs the row already on the table.

"How many to measure this?" (Pointing to the line marked 13.)

"Thirteen."

Teacher again prolongs the row with 13 more pieces taken from the supply.

"Who can write the story?"

John is sent to the board and writes,

$$\begin{array}{r} 8 \\ 5 \\ +13 \\ \hline \end{array}$$

Then the computation is performed.

All of the computations may now be performed upon linear magnitude.

REVIEW DRILLS

1. Add these columns:

(1)	(2)	(3)	(4)	(5)
564	59	92	76	59
27	74	74	83	64
84	65	68	29	72
13	48	53	94	89
+29	+56	+79	+37	+36

Every day one or more examples like the above should be placed on the blackboard for practice.

2. Dictate numbers to be written. Afterwards they should be read. Let the naughts be accounted for. Numbers should not be higher than 800.

UNCLASSIFIED PROBLEMS

1. 18 shirts were sold to 6 men. How many was that apiece if all bought the same number?

2. A lady made sashes for her three little girls. It took 36 inches of ribbon for one sash, 29 for another, and 42 for the third. How many inches of ribbon were used for the three sashes?

3. George's flower bed is 27 feet long, and William's is 16 feet long. How many feet longer is George's flower bed than William's?

4. My brother was away from home 15 weeks. He spent the time in 5 cities, staying the same length of time in each. How long was he in each city?

5. A little girl cut a piece of ribbon into 9 smaller pieces. Each of these pieces was 5 inches long. How many inches long was the large piece she had at first?

6. A man walked 8 miles a day. How many days did it take him to walk 40 miles?

7. It took Mr. Bain and his son 7 days to build a fence around a certain field. How many such fields could they fence in 63 days?

8. A little girl used 21 inches of baby ribbon to trim her doll's hats. It took 7 inches of ribbon for each hat. How many hats had the doll?

9. A lady went from Portland to California, spending 9 months in San Francisco, and 7 months in Los Angeles. She then returned to Portland. How long had she been away?

10. Mary studied her grammar 29 minutes, her geography 42 minutes, and her arithmetic 37 minutes. How many minutes did she study altogether?

11. John bought 5 bantam chickens, and then had 14. How many had he before buying the 5?

12. A dealer sold 42 handkerchiefs in boxes which held 6 each. How many boxes of handkerchiefs were sold?

13. 48 pecks of oats were divided equally among 4 bins. How many pecks were in each bin?

14. In a tobacco store were three cans holding snuff. One held 42 ounces, one 39 ounces, and one 28 ounces. How many ounces of snuff were in the store?

15. It took 3 yards of cloth to make a cloak. How many yards would it take to make 5 cloaks of the same size?

16. Martha's tape measure is 24 feet long, Lillian's 34 feet long, and Jane's 18 feet long. How long are all the tape measures, if put together?

17. A water tank when full holds 927 gallons of water. It now contains 856 gallons of water. How much does it lack of being full?

18. There were 54 yards of rubber hose used in watering a park. If there were 9 yards at each hydrant, how many hydrants were there?

19. Mr. Winton has 937 sheep. Mr. Randall has 247 sheep more than Mr. Winton. How many sheep has Mr. Randall?

LESSON NINETEEN

CARRYING IN SUBTRACTION

I. EXPLANATION. *Problem.* John had 83 pigeons, and sold 47 of them. How many were left?

Teacher says, "I would have to count 83 pigeons from the supply (writes 83), and then count 47 from the 83" (completing the expression),

$$\begin{array}{r} 83 \\ -47 \\ \hline \end{array}$$

"Can we take 7 ones from 3 ones?"

"No."

"Then let us take one of the eight tens and add it to the 3 ones. What will that make?"

"Thirteen."

"7 from 13 leaves what?"

"Six."

Teacher writes 6 below the line, and under the 7.

"I have taken one ten from eight tens, and have four more to take. How many tens will then be taken from the eight?"

"Five."

"And how many will be left?"

"Three."

Teacher completes the expression,

$$\begin{array}{r} 83 \\ -47 \\ \hline 36 \end{array}$$

NOTE. Teachers who prefer may, of course, obtain the 3 tens in the above remainder by subtracting 4 from 7 instead of 5 from 8. There is no difference in the philosophy of the two methods, and but little choice in the mechanical operations. In either case pupils should not be required to give the explanation. It is well enough for the teacher to give it, in order to justify the mode of procedure to the child. What the pupil needs is the ability to perform the operation promptly and correctly.

II. Drill on the following:

(1)	(2)	(3)	(4)	(5)
72	35	143	245	311
−58	−26	−69	−116	−109

(6)	(7)	(8)	(9)	(10)
204	300	701	563	710
−195	−84	−246	−418	−560

REVIEW DRILL

1. Add the following (one each day):

(1)	(2)	(3)	(4)	(5)
46	59	48	59	97
53	68	56	26	36
87	27	79	34	52
94	35	24	80	78
+36	+62	+56	+27	+94

2. Dictate numbers to be written. Afterwards have them read.

Let those requiring naughts occur frequently, and see that pupils understand the office of the naught.

Write no numbers higher than 900.

UNCLASSIFIED PROBLEMS

1. A boy cut a string into 8 top cords. Each of the cords was 9 inches long. How many inches long was the string at first?

2. A plumber worked at his trade in Boston 32 months. While there he worked the same length of time in each of 4 shops. How many months did he work in each?

3. A merchant bought four hogsheads of molasses. One contained 59 gallons, one 63 gallons, one 61 gallons, and one 58 gallons. How many gallons of molasses did he buy?

4. Our grocer delivered 28 pounds of butter, 4 pounds to each family. How many families did he supply?

5. William and Charles ran a race. It was 115 yards from the starting place to the goal. William was ahead of Charles, but fell down when he was 22 yards from the goal. How far had he run when he fell?

6. A class took 6 minutes to work each problem. How long did it take to work 4 problems?

7. A board was cut into 3 shelves. Each shelf was 5 feet long. How long was the board?

8. Mrs. Lovejoy raised 153 chickens in one year. This was 49 chickens more than Mrs. Simmons raised.

How many chickens did Mrs. Simmons raise that year?

9. There were 215 gallons of water in one tank, and 197 gallons more in another tank. How many gallons were in the second tank?

10. A sack holds 8 pecks of chestnuts. How many pecks will 4 such sacks hold?

11. From one tree 213 apples were picked. 173 apples were picked from another tree. How many more apples were picked from the first tree than from the second?

12. William had 35 minutes to work 7 problems in arithmetic. How much time was that for each problem?

13. A man worked 3 days digging a ditch. Each day he dug 9 feet of the ditch. How long was it?

14. A man pumped 37 quarts of water into a barrel in order to fill it. If it holds 120 quarts, how much water was in it when he began to pump?

15. A farmer put 3 bushels of carrots into each of 5 cow troughs. How many bushels did he feed his cows?

16. After 27 pecks of oats had been fed to a horse, there remained in the bin 38 pecks. How many pecks were in the bin at first?

17. A night watchman passed from one building to another for 12 hours each night. He spent altogether 2 hours in each building. How many buildings did he watch?

18. Mr. Brown bought 37 ounces of clover seed and 24 ounces of grass seed. How much more clover seed than grass seed did he buy?

19. A strip of wood 63 inches long was used in making rulers 9 inches long. How many rulers did it make?

20. A family used 93 pints of milk during the month of March. During April they used 17 pints more than in March. How many pints of milk did they use in April?

21. In Madison school there are 389 girls. The number of boys is 78 less than the number of girls. How many boys in the school?

22. Walter's grandmother is 67 years old. His grandfather is 9 years older than his grandmother. How old is Walter's grandfather?

23. 24 yards of braid were used on 3 cloaks. The length of the braid was the same on all the cloaks. How many yards were used on each cloak?

24. Henry weighs 73 pounds. Samuel weighs 67 pounds. How much more does Henry weigh than Samuel?

25. 24 peaches were divided equally among 6 children. How many did each receive?

26. Mr. Smith walked 21 miles in 7 hours. He walked the same distance every hour. How many miles did he walk each hour?

27. A man gave 12 cherries to each of 5 boys. How many did he give to all of them?

LESSON TWENTY

"TWO TIMES" AND "TIMES TWO"

I. Get from the multiplication table on page 360:

$2 \times 1 = 2$	$1 \times 2 = 2$
$2 \times 2 = 4$	$2 \times 2 = 4$
$2 \times 3 = 6$	$3 \times 2 = 6$
$2 \times 4 = 8$	$4 \times 2 = 8$
$2 \times 5 = 10$	$5 \times 2 = 10$
$2 \times 6 = 12$	$6 \times 2 = 12$
$2 \times 7 = 14$	$7 \times 2 = 14$
$2 \times 8 = 16$	$8 \times 2 = 16$
$2 \times 9 = 18$	$9 \times 2 = 18$
$2 \times 10 = 20$	$10 \times 2 = 20$
$2 \times 11 = 22$	$11 \times 2 = 22$
$2 \times 12 = 24$	$12 \times 2 = 24$

Notice that $2 \times 4 = 4 \times 2$, etc.

Learn to say the products in order from 2 to 24.

Drill on product and one factor, as follows:

$14 = ? \times 2$ or $2)\overline{14}$

$?$

$14 = 2 \times ?$ or $\frac{1}{2}$ of $14 = ?$

Notice that if the multiplier is missing, the division is measurement.

If the multiplicand is missing, the division is partitive.

II. DRILLS: (a) Every day put on the blackboard a number of the above combinations in miscellaneous order. Point to one and have a pupil at once name the product.

(b) Have the combinations on cards. Show one and have a designated pupil at once give the product.

(c) Have oral drills in which teacher names factors, and a designated pupil the product.

There are many other devices in use among teachers, the end sought by them all being to establish firmly mental connections between the various factors and their products.

Only persistent drill will do this.

Concert drills are of little or no value.

As soon as the products are known, they may be erased from the blackboard; and thereafter all problems in which two is a factor must be solved without consulting a table.

REVIEW DRILLS

1. Every day place on the board two columns of five figures each, that pupils may have practice in adding.

Thus,

$$\begin{array}{r} 67 \\ 94 \\ 82 \\ 37 \\ +54 \\ \hline \end{array}$$

2. Dictate numbers to be written not higher than 1000. Observe the directions previously given.

UNCLASSIFIED PROBLEMS

1. In a dry goods store, I saw 14 yards of blue ribbon; 27 yards of pink ribbon; 43 yards of white ribbon, and 57 yards of black ribbon. How many yards of ribbon did I see?

2. A boy found 5 nests in the barn, with 2 eggs in each. How many eggs did he find?

3. A farmer had 123 sheep. He sold 72 of them. How many had he left?

4. A street car made 4 trips over a car line and in doing so traveled 48 miles. How many miles did it travel each trip?

5. It took Martha 3 minutes to solve a problem. How long would it take her to solve 4 problems, if one takes as long as another?

6. A lady bought 20 yards of sheeting. If she used 5 yards for each pair of sheets, how many pairs did she make?

7. A merchant sold to one customer 84 feet of rope; to another 106 feet; to a third 95 feet, and then had 64 feet remaining. How many feet of rope had he at first?

8. It takes 5 inches of ribbon to make a bow. How many inches will it take to make 7 such bows?

9. In one flock there were 916 sheep. In another 349. How many more sheep were in the first flock than in the second?

10. 14 pints of milk were divided equally between 2 families. How many pints did each family receive?

11. Warren has a new set of books. Each book is 7 inches long. Placed end to end they reach 56 inches. How many books in the set?

12. A fire burned 295 pounds of wheat. 654 pounds were not harmed. How many pounds were there before the fire?

13. On Elm Street were 9 families. Each of them took 2 quarts of milk. How much milk did the milkman deliver on Elm Street?

14. Some workmen worked 63 days building 7 small houses. How many days was that for each house, if each took the same number of days?

15. Each of 2 bags contained 9 pounds of sugar. How many pounds were in both bags?

16. Jennie had 42 inches of silk ribbon. She used 26 inches of it to trim her doll's dresses. How many inches of ribbon had she left?

17. At a children's party, 2 ounces of candy were placed on each child's plate. If it took 14 ounces of candy, how many children were at the party?

18. In a month we fed 18 bushels of oats to 9 horses. How much was that for each horse, if all were fed alike?

19. There are 365 days in a year. There are 91 days in the three fall months. How many days are in the rest of the year?

20. Each of our cows gave 2 gallons at a milking. We got altogether 20 gallons. How many cows had we?

LESSON TWENTY-ONE

(a) AN UNSTATED CONDITION IN THE MULTIPLICATION OF A NATURAL PLURALITY

I. EXPLANATION. *Problem.* How many shoes have 9 horses?

"How many shoes would you count from the supply for one horse?"

"Four."

"How many times would you count 4?"

"Nine times."

"Write it."

Pupils write, $9 \times 4 =$

Pupils will notice that the fact that one horse has 4 shoes is a condition of the problem that is not stated.

(b) MONEY UNITS

II. EXPLANATION. *Problem.* A boy had 25 cents, earned 10 cents, and found 5 cents. How many cents had he then?

Probably the mode of computation will be obvious. If not, counters may be used, getting,

$$\begin{array}{r} 25 \\ 10 \\ +\ 5 \\ \hline 40 \end{array}$$

At first care should be taken to employ the same money unit throughout the problem, so that there will be no unstated conditions.

Thus the above problem would be less desirable at this stage, if it were written as follows:

A boy had a quarter, earned a dime, and found a nickel. How much money had he then?

REVIEW DRILLS

1. Continue drill on "two times" and "times two."
2. Dictate numbers not higher than 1000, to be written and afterwards read.
3. Addition of two columns.
4. Subtract,

$$\begin{array}{r} 902 \\ -154 \\ \hline \end{array}$$

UNCLASSIFIED PROBLEMS

1. Each of the houses in a certain block gave a gardener an order for 7 hollyhocks. It took 63 hollyhocks to fill the orders. How many houses in the block?
2. We added 17 feet to our garden hose. It was then 35 feet long. How long was it at first?
3. A farmer used 17 gallons of spray on his apple trees; 21 gallons on his pear trees, and 14 gallons on his peach trees. How many gallons did he use altogether?
4. A man's life is said to be divided into periods of 7 years. How many such periods has a man 42 years old passed through?
5. 80 yards of canvas are used for 8 tents of the same size. How many yards are used for one tent?

6. A man dug 20 pecks of potatoes from his garden on Monday; on Tuesday he dug 15 pecks; on Wednesday 31 pecks; on Thursday 17 pecks, and on Friday 9 pecks. How many pecks did he dig altogether?

7. 90 bushels of wheat were raised on a field of 9 acres. How many bushels is that to the acre?

8. A boy took 18 cents from his bank and left 24 cents. How many cents were in the bank at first?

9. 56 ounces of candy were placed in 7 bags. How many ounces were in each bag, if all the bags had the same quantity?

10. How long did it take Mr. Rollins to earn 35 dollars, if he earns 5 dollars a day?

11. What was received for 7 coats at 9 dollars apiece?

12. 20 pecks of meal were used in fattening 4 ducks. How much meal was that for each duck, if all were fed the same number of pecks?

13. How many fingers in 6 gloves?

14. We lived in our last house 54 months, and in our present house we have lived 17 months less. How long have we lived in our present house?

15. Ella has 6 books. Each of the books is 9 inches long. If she lays them end to end on the library table, how long a row will they make?

16. 97 bushels of potatoes were dug in September; 87 bushels in October; 46 bushels in November, and

96 bushels in December. How many bushels were dug in the four months?

17. A farmer had one churn which held 52 quarts of cream, and another which held 28 quarts less. How many quarts did the smaller churn hold?

18. At Harley school the running course is 90 yards long. The one at Columbia Park is 135 yards longer. How long is the course at Columbia Park?

19. Every time Charles visited his grandmother he stayed 9 days. When he had spent 63 days with her, how many visits had he made?

20. A farmer sold 84 cows to 7 different buyers. If all the buyers purchased the same number of cows, many did each receive?

21. A wagon cost 85 dollars. A horse cost 220 dollars. How much less did the wagon cost than the horse?

22. In the freight yard were five trains. One had 27 cars; one 32 cars; one 35 cars; one 16 cars; and one 43 cars. How many cars were in the freight yard?

23. At 8 cents a quart, what will 12 quarts of milk cost?

24. 49 cents were divided equally among 7 boys. How many cents did each boy receive?

25. In a month George spelled 149 words. 57 of them were spelled wrong. How many were correct?

26. A watch was bought for 15 dollars and sold for 9 dollars more. What was it sold for?

27. 9 umbrellas were sold for 3 dollars each. How much was received for them?

LESSON TWENTY-TWO

"THREE TIMES" AND "TIMES THREE"

I. EXPLANATION. Get from the multiplication table the following:

$3 \times 1 = 3$	$1 \times 3 = 3$
$3 \times 2 = 6$	$2 \times 3 = 6$
$3 \times 3 = 9$	$3 \times 3 = 9$
$3 \times 4 = 12$	$4 \times 3 = 12$
$3 \times 5 = 15$	$5 \times 3 = 15$
$3 \times 6 = 18$	$6 \times 3 = 18$
$3 \times 7 = 21$	$7 \times 3 = 21$
$3 \times 8 = 24$	$8 \times 3 = 24$
$3 \times 9 = 27$	$9 \times 3 = 27$
$3 \times 10 = 30$	$10 \times 3 = 30$
$3 \times 11 = 33$	$11 \times 3 = 33$
$3 \times 12 = 36$	$12 \times 3 = 36$

Notice that $3 \times 2 = 2 \times 3$, etc.

Learn to say the products in order from 3 to 36.

Drill on product and one factor as follows:

$21 = 3 \times ?$, or $\frac{1}{3}$ of $21 = ?$

$21 = ? \times 3$, or $3\overline{)21}$.

$?$

II. DRILL. Observe what was said about drill under "two times" and "times two."

There should now be daily drill on both tables. As soon as the products are known they should be erased from the table on the blackboard.

REVIEW DRILLS

1. Add daily one example like this,

$$\begin{array}{r} 456 \\ 742 \\ 964 \\ +735 \\ \hline \end{array}$$

2. Subtract daily with carrying, thus,

$$\begin{array}{r} 506 \\ -264 \\ \hline \end{array}$$

UNCLASSIFIED PROBLEMS

1. A cow gave 9 quarts of milk every day. How much milk did she give in 4 days?

2. Henry Jones worked in his garden 13 hours one week, 9 hours another, 15 hours another, and 17 hours another. How many hours did he work in his garden during the four weeks?

3. In the reading class, 7 pupils were heard in 14 minutes. How long did each pupil read, if the time was the same for all?

4. Mary bought 9 oranges at 3 cents apiece. How much did she pay for them?

5. A woman ironed 14 napkins, 18 sheets, and 26 towels. How many pieces did she iron?

6. A company of soldiers when full consists of 84 men. If a certain company needs 27 more men to fill it, how many men are now in the company?

7. How many shoe strings were needed for 8 pairs of shoes?

8. It took 27 yards of carpet for one of the bedrooms of a house. The parlor needed 21 yards more than the bedroom. How many yards of carpet did the parlor take?

9. It required 27 yards of sidewalk for the pavement in front of 3 stores of the same width. How much was that for each store?

10. If 2 pints of milk are used in making a pudding, how many puddings will 18 pints make?

11. A grocer had on hand 256 pounds of sugar, and sold 198 pounds. How many pounds had he left?

12. A boy bought 2 tops for 8 cents apiece. What did he pay for them?

13. Mr. Wilson paid 45 dollars for a set of books, at 5 dollars apiece. How many books in the set?

14. There were 12 roses on one bush; 15 on another; 13 on another; 17 on another, and 24 on a fifth. How many roses were on the five bushes?

15. At 5 cents apiece, how many oranges can you buy for 30 cents?

16. 194 oranges were sold from one cart, and 97 oranges from another. How many more oranges were sold from the first cart than from the second?

17. 5 boys picked 30 quarts of cranberries. How many quarts was that apiece, if all picked the same quantity?

LESSON TWENTY-THREE

"FOUR TIMES" AND "TIMES FOUR"

I. EXPLANATION. Get from the multiplication table the following:

$4 \times 1 = 4$	$1 \times 4 = 4$
$4 \times 2 = 8$	$2 \times 4 = 8$
$4 \times 3 = 12$	$3 \times 4 = 12$
$4 \times 4 = 16$	$4 \times 4 = 16$
$4 \times 5 = 20$	$5 \times 4 = 20$
$4 \times 6 = 24$	$6 \times 4 = 24$
$4 \times 7 = 28$	$7 \times 4 = 28$
$4 \times 8 = 32$	$8 \times 4 = 32$
$4 \times 9 = 36$	$9 \times 4 = 36$
$4 \times 10 = 40$	$10 \times 4 = 40$
$4 \times 11 = 44$	$11 \times 4 = 44$
$4 \times 12 = 48$	$12 \times 4 = 48$

Notice that $4 \times 2 = 2 \times 4$, etc.

Notice that the combinations enclosed in the rectangle have already been learned.

Learn to say the products in order from 4 to 48.

Drill on product and one factor as follows:

$24 = 4 \times ?$ or $\frac{1}{4}$ of $24 = ?$

$24 = ? \times 4$ or $4)\overline{24}$

$?$

II. DRILL.

Drill on these tables in connection with those previously learned.

When the combinations are known they should be erased from the rectangular table on the blackboard.

REVIEW DRILL

1. One abstract example in addition should be given each day.

2. One abstract example in subtraction should be given each day.

3. If these problems are dictated and afterwards read, it will give sufficient practice in writing and reading numbers.

CLASSIFIED PROBLEMS

1. A sailor was 15 years at sea, spending 5 years on each vessel. On how many vessels did he sail?

2. George bought a baseball for 42 cents, a bat for 38 cents, and a glove for 75 cents. What did he pay for the three?

3. A yardstick is 3 feet long. How long is the wood used in 4 yardsticks?

4. A tank holding 36 gallons of oil was emptied into cans holding 4 gallons each. How many cans were used?

5. Roger had 344 cents saved up for Christmas. After Christmas he had 17 cents. How much did he spend?

6. Each of 8 boys had a knife. Their knives contained altogether 40 blades. If the knives were alike, how many blades were in each?

7. A strip of sole leather 36 inches long was cut into pieces 9 inches long, each piece being enough for the sole of one shoe. How many shoes could be soled with the leather?

8. A man bought 6 postage stamps in one sheet for 24 cents. How many cents did each stamp cost?

9. Four boys have fish lines. One is 19 feet long; one 26 feet; one 15 feet, and one 32 feet. How many feet of string are in all four of the fish lines?

10. A boy earns 6 dollars a week. How long will it take him to earn 18 dollars?

11. May has to practice on the piano 52 hours each month. This month she has still 28 hours to practice. How many hours has she already practiced?

12. How many ears have 6 rabbits?

13. In 8 equal boxes of tooth powder there were 16 ounces. How many ounces were in each box?

14. After selling 65 yards from a piece of cloth, a merchant had 17 yards of the piece remaining. How many yards were in the piece at first?

15. How long did it take a doctor to see 8 patients if he gave each one 5 minutes?

16. 7 pairs of shoes of the same price were sold for 28 dollars. How much was that a pair?

17. Edward had 27 marbles; William 32; Howard 46; James 62, and Walter 53. How many marbles had all the boys?

18. There were 967 soldiers in a camp. 549 of them returned home at the end of two weeks. The others remained a month. How many remained?

19. How many legs have 8 common chairs?

20. A creamery had for sale in the morning 287 pint bottles of cream. At night 98 bottles were left. How many pints were sold during the day?

21. How many cords 8 inches long can be made from a string 32 inches long?

22. At the beginning of winter a family bought 24 pecks of potatoes. They have used 18 pecks. How many pecks are left?

23. A blacksmith used an iron rod 27 feet long to make 3 equal wagon tires. How many feet of the rod did he use for each tire?

24. 9 hats were sold for 4 dollars apiece. What was received for all of them?

25. It required 7 yards of cloth to make a suit. How many such suits could be made from a piece of cloth containing 21 yards?

26. A barrel contained 121 quarts of vinegar. 37 quarts leaked away. How many quarts were left in the barrel?

27. After buying 35 new books, a man had 302 books in his library. How many books had he at first?

LESSON TWENTY-FOUR

TO MULTIPLY WHEN THE MULTIPLICAND IS LARGER THAN 12

I. EXPLANATION. Find 4×248.

Write it thus,

$$\begin{array}{r} 248 \\ 4\times \\ \hline 992 \end{array}$$

$4 \times 8 = 32$. Put down the right hand figure 2 and carry 3.

$4 \times 4 = 16$, and 3 carried makes 19. Put down the right hand figure 9, and carry 1.

$4 \times 2 = 8$, and 1 carried makes 9. Put down the 9.

Therefore, $4 \times 248 = 992$.

It will be noticed that, in this solution, the pupil has to add 3 to 16, which is not one of the combinations of the addition table. It is, therefore, necessary that the pupil at this time should take up an extension of the addition table which he needs to use in multiplication and will come gradually to use in addition, except when the columns are too high.

Take any of the combinations as,

$$\begin{array}{rrrrrcr} 1 & 1 & 1 & 1 & 1 & & 1 \\ +1 & +11 & +21 & +31 & +41 & \text{etc., to,} & +91 \\ \hline 2 & 12 & 22 & 32 & 42 & & 92 \end{array}$$

4	4	4	4		4
+6	+16	+26	+36	etc., to,	+96
10	20	30	40		100

5	5	5		5
+7	+17	+27	etc., to,	+87
12	22	32		92

That is, the right hand figure of the original combination remains unchanged, no matter how many times 10 is added to either addend.

II. DRILL.

48	58	29	
2×	3×	4×	etc.

Dictate examples like these until the process is known.

REVIEW DRILLS

1. Every day extend two of the combinations of the addition table.

2. Write and read numbers to 1000.

UNCLASSIFIED PROBLEMS

1. Mr. Williams was ill for 57 months. This was 19 months longer than he afterwards spent in travel. How long did he travel?

2. 28 dollars were received for 7 pairs of shoes of the same kind. How much was that a pair?

3. At 6 cents apiece, what will 8 lead pencils cost?

4. While playing on the beach Lucy found 134 seashells; Mary found 193 shells, and Jane found 276. How many seashells did the three girls find?

5. William and James ran a race of 220 yards. When William had finished, James had run only 197 yards. How far did William beat James?

6. A lady spent 35 cents for 5-cent postage stamps. How many did she buy?

7. The curtains for 6 windows were made of 12 yards of lace. How many yards was that for each window?

8. Mr. Willis bought 4 cows for 48 dollars apiece. What did they cost him?

9. 35 gallons of wine will fill how many 5-gallon kegs?

10. John had a ball of twine 256 inches long. He used 198 inches tying up bundles. How many inches of twine were left?

11. 3 bags of potatoes weigh 56 pounds apiece. What do they all weigh?

12. It required 54 inches of picture wire to hang 6 pictures. How many inches was that for each picture?

13. A newsboy sold 56 papers on Monday, 48 on Tuesday, 37 on Wednesday, 84 on Thursday, and 63 on Friday. How many papers did he sell in the five days?

LESSON TWENTY-FIVE

AN UNSTATED CONDITION IN THE MEASUREMENT OF NATURAL PLURALITY

I. EXPLANATION. *Problem.* 24 legs will be enough for how many common chairs?

"What will you do first?"

"Count 24 legs from the supply."

"Then what?"

"Count out 4 at a time for each chair, until all are gone."

"Write it."

Pupils write, $4\overline{)24}$

6

Notice the unstated condition, "one common chair has 4 legs."

II. DRILL.

Point out the unstated conditions in the following:

(1) How many hogs must be slaughtered to make 16 sides of bacon?

(2) How many gloves have altogether 30 fingers?

(3) A boy had 40 cents in nickels. How many nickels had he?

(4) 32 wheels are enough for how many wagons?

(5) How many birds have altogether 24 wings?

REVIEW DRILLS

1. Every day extend three of the combinations of the addition table.

2. Dictate one or two like the following each day:

46	57	89
3 ×	4 ×	2 ×

UNCLASSIFIED PROBLEMS

1. Mr. Johnson spent 78 dollars for a horse, and had 256 dollars remaining. How many dollars had he before buying the horse?

2. A doctor received 4 patients in 24 minutes. How many minutes did he give to each?

3. If 3 pieces of string each 8 feet long are laid end to end, how many feet long will they be?

4. How many common tables must be made to use 36 legs?

5. Margaret had a vacation of 45 days, while Elizabeth's vacation was only 36 days. How much longer was Margaret's vacation than Elizabeth's?

6. A family lived 21 years in 3 different houses. How long was that in each house, if the time was the same for each house?

7. 8 shoe laces are enough for how many pairs of shoes?

8. A steamboat carried 24 passengers one trip, 42 another, 15 another, 37 another, and 51 on the fifth. How many passengers did the boat carry on the five trips?

9. A boy had 9 cents in his pocket. In his bank were 22 cents more than in his pocket. How many cents were in his bank?

10. A glove maker finished 60 fingers. How many gloves did he make?

11. A ball and a top cost together 113 cents. The ball cost 98 cents. What did the top cost?

12. A grocery store filled 9 orders for 4 pounds of sugar each. How many pounds did it take to fill these orders?

13. A footpath led across four fields. Across the first field the path was 73 yards long; across the second 67 yards long; across the third 84 yards, and across the fourth 48 yards. How long was the path?

14. A baker bought a horse for 225 dollars, which was 37 dollars more than his wagon cost. What did he pay for the wagon?

15. A strip of wood 32 inches long was cut into 4 equal rulers. How long was each ruler?

16. How many horses will need 28 shoes?

17. Mr. Roberts took the train from his home in Marston, and rode to Trent in 18 hours. He remained in Trent 36 hours, and then rode on to Weston in 15 hours. He stayed 12 hours at Weston. Then he rode back to his home at Marston in 33 hours. How many hours was he away?

18. If 48 flowers are made up into bunches of 6 flowers each, how many bunches will there be?

19. A man earned 238 dollars in a month. He spent 149 dollars. How much did he save?

20. A man gave 7 apples to each of 9 children. How many did he give to them all?

LESSON TWENTY-SIX

"FIVE TIMES" AND "TIMES FIVE"

I. EXPLANATION. Get from the multiplication table:

$5 \times 1 = 5$	$1 \times 5 = 5$
$5 \times 2 = 10$	$2 \times 5 = 10$
$5 \times 3 = 15$	$3 \times 5 = 15$
$5 \times 4 = 20$	$4 \times 5 = 20$
$5 \times 5 = 25$	$5 \times 5 = 25$
$5 \times 6 = 30$	$6 \times 5 = 30$
$5 \times 7 = 35$	$7 \times 5 = 35$
$5 \times 8 = 40$	$8 \times 5 = 40$
$5 \times 9 = 45$	$9 \times 5 = 45$
$5 \times 10 = 50$	$10 \times 5 = 50$
$5 \times 11 = 55$	$11 \times 5 = 55$
$5 \times 12 = 60$	$12 \times 5 = 60$

The combinations enclosed in the rectangle have already been learned.

Notice that $5 \times 2 = 2 \times 5$, etc.

Learn the products in order from 5 to 60.

Drill on product and one factor as follows:

$5 \times ? = 20$ or $\frac{1}{5}$ of 20 = ?

$? \times 5 = 20$ or $5\overline{)20}$

?

II. DRILL on these combinations in connection with those already learned.

When the combinations are known they should be erased from the rectangular table on the blackboard.

REVIEW DRILLS

1. Every day extend 3 combinations of the addition table.

2. Dictate every day one abstract problem in addition, one in subtraction, and one in multiplication by 3, 4, or 5.

UNCLASSIFIED PROBLEMS

1. How many gloves will it take for 7 children?

2. 40 inches of gold chain were used by a jeweler in making 4 necklaces of equal length. How many inches of chain in each necklace?

3. William picked 74 quarts of berries. James picked 39 quarts more than William. How many quarts did James pick?

4. Harry bought 3 new tops, paying 8 cents apiece. How many cents did he pay for them?

5. Henry bought 27 fish hooks. He then had 40. How many had he at first?

6. Robert found 4 birds' nests with 24 eggs in them. How many eggs in each nest, if all had the same number?

7. Harry's whip handle is 3 feet long. How long must a stick be which will make 5 such whip handles?

8. How many quarts of milk each week were bought by a family that used 4 quarts a day?

9. A boy dug 49 pecks of potatoes from one patch; 37 pecks from another; 43 from another; and 57 pecks from the fourth. How many pecks did he dig from the four patches?

10. Mr. Jones spent 18 months visiting some of the large cities of the East. If he spent 3 months in each city, how many did he visit?

11. A narrow strip of leather 10 feet long was cut into 5 equal top cords. How long was each cord?

12. George's kite string was 103 feet long. William's was 87 feet long. How much longer was George's string than William's?

13. A book dealer sold 75 geographies, and still had 89. How many had he at first?

14. Samuel's father gave him 49 cents. He then had 107 cents. How many cents had he at first?

15. A party of boys got on a street car. Their leader paid the conductor 40 cents fare. If the fare was 5 cents each, how many boys were in the party?

16. Mr. Howland earned 39 dollars a week. This was 15 dollars less than Mr. Randall earned. How much did Mr. Randall earn in a week?

17. 9 sacks of wheat contained 3 bushels each. How many bushels were in all the sacks?

18. If oranges were sold for 2 cents apiece how many were sold for 24 cents?

19. What did 5 tons of coal cost at 9 dollars a ton?

LESSON TWENTY-SEVEN

TO READ AND WRITE NUMBERS OF TWO PERIODS

I. EXPLANATION. Numbers are composed of periods of three figures each.

Pupils already know how to read and write numbers of one period, and are now to learn to read and write those of two periods.

Each period of three figures has a name. The name of the first or right hand period is "units" or "ones." Thus,

298 means 298 units or ones.

57 means 57 units or ones.

In reading this first period, its name is omitted.

The name of the second period is "thousands." It is always given in reading numbers.

Thus, 298,157 is read, 298 thousands, 157.

II. DRILL. At first, until pupils are familiar with the names of the periods, it is well to write these names above the numbers thus:

	Thousands	~~Units~~	
(1)	467,	284	= 467 thousands, 284.
(2)	40,	280	= 40 thousands, 280.
(3)	506,	057	= 506 thousands, 57.
(4)	509,	006	= 509 thousands, 6.
(5)	400,	000	= 400 thousands.

In the above diagram, units, the name of the first period, has a line drawn *through it*, because it is not spoken in reading the number.

In (3), 0 is placed before the 57 in order that the first period may have three figures. If it were omitted the number when not in the diagram would appear thus, 50657, which puts the 6 in the first period.

In (4), two naughts are placed before 6 for the same reason; and in (5) since there are no units, three naughts appear in the first period.

Pupils should learn:

First, to read numbers in the diagram.

Second, to account for the naughts.

Third, to read numbers not in the diagram.

Fourth, to write numbers in the diagram.

Fifth, to write numbers not in the diagram.

REVIEW DRILLS

1. Extension of addition table, 3 combinations daily.

2. Miscellaneous drill on multiplication table to 5's.

UNCLASSIFIED PROBLEMS

1. Add:

$$\begin{array}{r} 165 \\ 29 \\ 74 \\ +58 \\ \hline \end{array}$$

2. Mr. Thomson earned 189 dollars in a month, and spent 94 dollars. How many dollars did he save in a month?

3. Henry had 95 cents saved for Christmas, and Edward 76 cents. How many cents did Henry save more than Edward?

4. A farmer sold 9 sheep at the same price, receiving 54 dollars. How much was that apiece?

5. $5 \times 183 = ?$

6. A farmer fed 15 bushels of turnips to his sheep in 5 days. How many bushels a day was that?

7. A robin stole 29 cherries from one tree, 16 from another, 32 from another, 20 from another, and 22 from a fifth. How many cherries did he get from the five trees?

8. In John's vegetable garden there were 9 equal rows of cabbages, with 54 cabbages in all of them. How many cabbages in each row?

9. $306 - 198 = ?$

10. $29 + 67 + 54 + 93 + 68 = ?$

11. Mr. Miller rents his house by taking a lease for 3 years. He has lived in the house 18 years. How many times has he taken the lease?

12. What will 125 cords of wood cost at 6 dollars a cord?

13. What is the cost of 7 city lots at 185 dollars each?

14. A builder used 8 loads of brick a day for 39 days. How many loads did he use altogether?

15. 108 dollars were spent for land at 12 dollars an acre. How many acres were bought?

16. A house contains 24 windows. In each window there are 8 panes of glass. How many window panes are in the entire house?

LESSON TWENTY-EIGHT

HOW TO MEMORIZE DRY MEASURE

I. EXPLANATION. Place on the table a vessel holding about two bushels of sand, and the following measures: pint, quart, peck, bushel.

Make pupils familiar with the names of the measures; then ask,

"How many pints will a quart measure hold?"

Let the answer be found by trial.

When it has been found, write on the board, 2 pints = 1 quart.

"How many quarts will a peck measure hold?"

After finding the answer by trial, write on board, 8 quarts = 1 peck.

"How many pecks will a bushel measure hold?"

After finding the answer by trial, write on board, 4 pecks = 1 bushel.

The table may now be memorized.

II. DRILL. Have the measures in sight during the exercise, and let the pupil touch both measures while giving the connecting number.

Thus, touching the quart (qt.) measure with one hand and the peck (pk.) with the other, he says, "8 qt. = 1 pk."

Again touching the peck (pk.) with one hand and the bushel (bu.) with the other, he says, "4 pk. = 1 bu."

REVIEW DRILL

1. Extend 3 combinations of the addition table each day.

2. Read and write numbers of two periods. (See Lesson 27.)

UNCLASSIFIED PROBLEMS

1. One tub contained 60 pounds of butter; another tub contained 39 pounds. How many pounds less in the second tub than in the first?

2. 5 wagons were drawn by 20 horses. How many horses to each wagon, if all the wagons had the same number of horses?

3. How many shoes will be needed for 5 horses?

4. $9036 - 5842 = ?$

5. It took Lucy 28 minutes to work 4 problems. If one problem took as long as another, how many minutes were given to each?

6. $5 \times 683{,}417 = ?$

7. How many oranges did Richard buy for 24 cents, if they were 4 cents apiece?

8. 4 curtains of the same size were made from 12 yards of cloth. How much cloth was used for each curtain?

9. How many days more in the three months of spring than in the three months of autumn?

10. 4 hats at the same price cost 12 dollars. How much was that apiece?

11. A boy earned 18 cents on Monday; 25 cents on Tuesday; 9 cents on Wednesday and 19 cents on Thursday. How much did he earn in the four days?

12. How many pages are there in 8 leaves of a book?

13. Lillian's father gave her some old letters with 3 stamps on each. If she got from them 21 stamps, how many letters were there?

14. A farmer received 42 dollars for some sheep. How many did he sell at 6 dollars apiece?

15. 108 apples were picked from one tree, and 97 from another. How many more were picked from the first tree than from the second?

16. A man can pick 42 bushels of apples a day. How many bushels can he pick in 5 days?

17. 314 small packages weigh 3 ounces apiece. How many ounces do they all weigh?

18. A basket holds 12 eggs. How many such baskets would be needed to hold 108 eggs?

19. A grocer had 2897 pounds of sugar. He sold 1598 pounds. How many pounds were left?

20. Mr. Thomson owns 8 city lots. On each of them he pays 43 dollars taxes. How much taxes does he pay on them all?

21. 9 sheep were sold for 54 dollars. How much was that apiece, if the price was the same for all?

22. At 6 dollars a pair, how many pairs of shoes were sold for 72 dollars?

LESSON TWENTY-NINE

TO SOLVE A PROBLEM REQUIRING AN OPERATION IN ADDITION AND ANOTHER IN SUBTRACTION

I. EXPLANATION. *Problem.* A grocer had 185 pounds of flour in one barrel; 167 pounds in another, and 179 pounds in another. If he sells 265 pounds, how many pounds will he have left?

"There are two problems here. Who can state the first one?"

After several attempts, it is stated by the teacher, as follows:

"A grocer had 185 pounds of flour in one barrel; 167 pounds in another, and 179 pounds in another. *How many pounds in all?*"

"What words have I used which are not in the problem on the board?"

"*How many pounds in all?*"

"Yes; whenever two problems are written together, some words are left out. You will have to put them in, in order to separate the problems."

"You may solve this first problem."

Class does so, as follows:

$$\begin{array}{r} 185 \\ 167 \\ +179 \\ \hline 531 \end{array}$$

State the second problem.

"*A grocer had 531 pounds of flour.* If he sold 265 pounds, how many pounds had he left?"

This is solved thus,

$$\begin{array}{r} 531 \\ -265 \\ \hline 266 \end{array}$$

To separate the problem requiring two operations into two problems, each requiring one operation, is the only new difficulty.

II. DRILL.

Write on the blackboard a problem requiring these two operations. Let a pupil state the first of the two problems into which it may be separated. Teacher gives the answer to this first problem without writing the numerical expression. Using this answer, another pupil states the second problem.

Continue this until problems are readily separated into their two parts without solving.

REVIEW DRILLS

1. Extend each day 3 combinations of addition table.

2. Read and write numbers of two periods.

3. Drill on dry measure. When a pupil forgets a connecting number send him to the measures.

UNCLASSIFIED PROBLEMS

1. A boy went out to shoot his toy cannon. He took two flasks of powder. One held 5 ounces, and the other 7. If he used only 6 ounces of powder, how many ounces did he bring back?

2. At a meeting 9 men used 27 minutes in making short speeches of the same length. How long did each speak?

3. A bottle of lemon extract held 3 pints. How much of the extract was there in 9 such bottles?

4. A year contains 365 days. When 187 days of the year have passed, how many days remain?

5. 8 barrels of potatoes contained 10 pecks in each barrel. How many pecks altogether?

6. Joseph spent 36 cents for 9 small pies. How much was that apiece?

7. $5608 + 1937 + 2463 + 987 = ?$

8. A little girl spent 36 minutes looking into 4 Christmas windows. If she spent the same time at each, how long did she look into each window?

9. $4 \times 35{,}971 = ?$

10. A girl had 9 pieces of candy on one plate, and 7 pieces on another. If she ate 6 pieces, how many pieces were left on both plates?

11. 32 quarts of plums are how many pecks, if there are 8 quarts in one peck?

12. There were 8 oranges in one dish and 7 in another. If a boy ate 4 of them, how many were left?

13. Chester had three rabbit hutches. In one there were 6 rabbits; in another 4 rabbits; and in the third 8 rabbits. One morning 9 of the rabbits were missing. How many were left?

14. $40{,}000 - 10{,}018 = ?$

LESSON THIRTY

"SIX TIMES" AND "TIMES SIX"

I. EXPLANATION. Get from the multiplication table the following:

$6 \times 1 = 6$	$1 \times 6 = 6$
$6 \times 2 = 12$	$2 \times 6 = 12$
$6 \times 3 = 18$	$3 \times 6 = 18$
$6 \times 4 = 24$	$4 \times 6 = 24$
$6 \times 5 = 30$	$5 \times 6 = 30$
$6 \times 6 = 36$	$6 \times 6 = 36$
$6 \times 7 = 42$	$7 \times 6 = 42$
$6 \times 8 = 48$	$8 \times 6 = 48$
$6 \times 9 = 54$	$9 \times 6 = 54$
$6 \times 10 = 60$	$10 \times 6 = 60$
$6 \times 11 = 66$	$11 \times 6 = 66$
$6 \times 12 = 72$	$12 \times 6 = 72$

The combinations enclosed in the rectangle have already been learned.

Notice that $6 \times 2 = 2 \times 6$, etc.

Learn the products in order from 6 to 72.

Drill on factor and product, as follows:

$6 \times ? = 18$ or $\frac{1}{6}$ of 18 = ?

$? \times 6 = 18$ or $6\overline{)18}$

?

II. DRILL on the combinations in connection with those already learned. When the combinations are known, they should be erased from the rectangular table on the blackboard.

REVIEW DRILLS

1. Extend each day 3 combinations of the addition table.
2. Read and write numbers of two periods.
3. Multiplication table to 6's.

UNCLASSIFIED PROBLEMS

1. A lady visited her mother 6 times in a year, and spent 9 days with her each visit. How many days did the lady spend with her mother in a year?
2. Three Sunday-school picnics went down the river on the same day. There were 247 children at one, 197 at another, and 273 at the third. How many children were at the three picnics?
3. 4 babies weighed 32 pounds. How much was that apiece, if each baby weighed the same as the others?
4. 20 quarts of cider were poured into jugs each holding 4 quarts. How many jugs were filled?
5. A workingman earned 8 dollars one week and 12 dollars the next. During the two weeks he spent 13 dollars. How many dollars had he left?
6. A dealer bought 85 quarts of oysters, and could sell only 79 quarts. How many quarts were not sold?

7. 24 horseshoes will be enough for how many horses?

8. It took 35 inches of lace for 7 doll dresses of the same kind. How many inches were used on each dress?

9. An electric car makes 8 trips a day. How many trips does it make in 7 days?

10. Some prize apples were displayed in boxes, 6 apples in each box. How many boxes were used to show 42 apples?

11. Mr. Williams had 7 rosebushes on his lawn, and planted 6 more. The frost killed 5 of them. How many rosebushes did he then have?

12. A peddler tied up 54 shoestrings in bunches, each of which contained 6 shoestrings. How many bunches were there?

13. Gertrude paid 56 cents for 7 hoops, all of the same price. How much was that apiece?

14. $1927 + 80{,}059 + 23{,}604 + 59{,}712 + 316 = ?$

15. $205{,}617 - 193{,}782 = ?$

16. $6 \times 58{,}408 = ?$

17. When a milkman finished delivering his milk he had 32 gallons left, 8 gallons in each of the cans that were not empty. How many of his cans were not empty?

18. A harness maker made 18 reins. These were enough for how many sets of harness?

19. $567 + 2943 + 759 + 1807 + 3715 = ?$

20. $50{,}000 - 11{,}118 = ?$

CHAPTER III

STEPS OF THIRD YEAR

LESSON THIRTY-ONE

HOW TO REFER TO THE MULTIPLICATION TABLE FOR INEXACT DIVISION BY 2

I. EXPLANATION. *Problem.* A lady had 9 apples. She wished to place as many of them as she could on 2 plates, so that an equal number would be put on each. How many apples were put on each plate?

Imagined counting gives the expression $\frac{1}{2}$ of 9 = ?

On the blackboard as much of the multiplication table as is needed should be reproduced.

Place the pointer at $\frac{1}{2}$ and move it to the right until you come to 8. Notice that the next number is 10 and that 9 does not appear in that row at all.

"8 and how many make 9?"

"One."

Write 1 as an exponent of 9 thus, $\frac{1}{2}$ of 9^1 =

Then pass from 8 upwards to 4, at the top of the column. Then complete the expression thus:

$$\frac{1}{2} \text{ of } 9^1 = 4$$

"There will be 4 on each plate, and one over."

In a similar manner the quotient and remainder may be found when the division is by measurement.

II. DRILL.

Complete such expressions as the following, at first by reference to the visible table, afterwards by reference to the memorized table.

$\frac{1}{2}$ of 7 =	2) 9	2)15
$\frac{1}{2}$ of 9 =	2)23	2)13
$\frac{1}{2}$ of 11 =	2) 5	2)19
$\frac{1}{2}$ of 19 =	2)21	2) 9
$\frac{1}{2}$ of 25 =	2)17	2)11

REVIEW DRILL

1. Extend each day 3 combinations of the addition table.
2. Drill daily on the multiplication table to 6's.
3. Occasionally read and write numbers.
4. Occasionally recite dry measure.

UNCLASSIFIED PROBLEMS

1. How many wheels will be needed for 9 wagons?
2. 19 horses will make how many teams, if there are 2 horses in each team?
3. Harry had 2 equal kegs into which he wished to put 9 gallons of cider. How much did he put in each keg if he filled them both, and there was 1 gallon over?
4. 207,864 + 109,207 + 89,842 + 5674 = ?
5. 50,004 − 46,003 = ?
6. 4 × 237,864 = ?

7. 15 ounces of camphor were to be put up in 2-ounce packages. How many packages did it make?

8. Samuel had two young peach trees. On one were 9 peaches and on the other 7. If Samuel picked 8 peaches, how many were left on both trees?

9. William lost 18 marbles and still had 87. How many had he at first?

10. A building of 54 rooms has 9 rooms on each floor. How many floors has the building?

11. A piece of ribbon 13 inches long was to be cut into 2 equal pieces. How long was each piece, if 1 inch of the ribbon was left over?

12. It is 9 yards from one side of a lot to the other. Bessie ran across the lot 7 times. How many yards did she run?

13. A number of boys had 7 chestnuts apiece, and 35 chestnuts altogether. How many boys were there?

14. On a farm were 725 sheep and lambs. 246 of these were lambs. How many old sheep were there?

15. It took 27 yards of carpet for one room of a house, 18 yards for another, 32 for another, 22 yards for a fourth, and 26 for a fifth. How many yards of carpet were used for the five rooms?

16. Each of 24 cows gave 6 quarts at a milking. How many quarts did they all give?

17. Henry earned 24 dollars, and then had 103 dollars. How many dollars had he at first?

18. $6 \times 14{,}837 = ?$

LESSON THIRTY-TWO

TO MEMORIZE LIQUID MEASURE AND THE TABLE OF WEIGHT

I. EXPLANATION. Place on the number table a pint measure, a quart measure, a gallon measure, and a vessel containing sand.

"How many pints will a quart measure hold?"

Let the answer be found by trial. Then write on the board, 2 pt. = 1 qt.

"How many quarts will a gallon measure hold?"

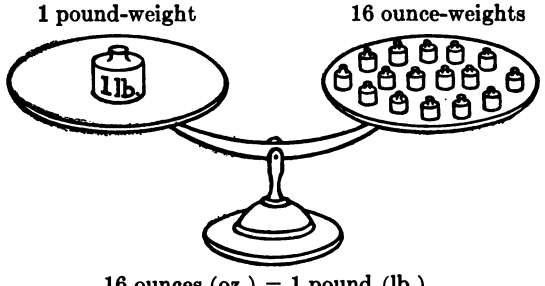
After finding the answer by trial write, 4 qt. = 1 gal.

The table may now be memorized.

Weight.

Place on the number table a pair of weighing scales, together with a number of ounce weights, and a pound weight.

Get by trial, 16 ounces = 1 pound.



16 ounces (oz.) = 1 pound (lb.)

"If we had scales large enough to hold a ton, how many pounds would it take to balance the ton."

Pupils can not answer, and teacher writes, 2000 pounds = 1 ton.

II. DRILL. Have the measures in sight during the exercise, and let the pupil touch both measures while giving the connecting number.

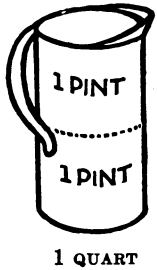
Thus, taking a pint (pt.) in one hand and a quart (qt.) in the other, he says,

"2 pt. = 1 qt."

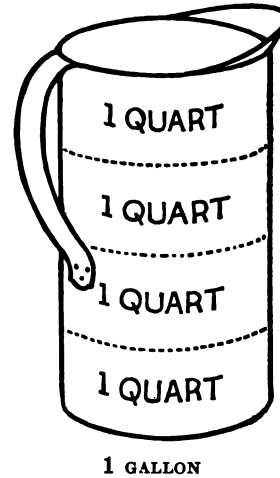
Taking a qt. in one hand and a gallon (gal.) in the other, he says,

"4 qt. = 1 gal."

Touching an ounce (oz.) with one hand and a pound (lb.) with the other, he says, "16 oz. = 1 lb."



1 QUART
2 pints (pt.) = 1 quart (qt.)



1 GALLON
4 qt. = 1 gallon (gal.)

Touching a pound with one hand and waving the other arm extended to indicate a large volume, he says,

"2000 lb. = 1 ton."

REVIEW DRILLS

1. Multiplication combinations to 6's, with corresponding divisions. When 2 is a factor let the divisions be inexact.

2. Occasional drills on weight, liquid measure, and dry measure.

UNCLASSIFIED PROBLEMS

1. Mr. Watson's income is 2810 dollars a year. This is 965 dollars less than Mr. Thorp's income. What is Mr. Thorp's income?

2. It is 19 miles from Harry's house to his uncle's. In 2 days Harry walked all the distance but 1 mile. How far did he walk each day, if he walked the same distance each day?

3. 17 pints of spruce beer will fill how many bottles, if each bottle holds 2 pints?

4. Mr. Stine bought 5 pounds of meat one week and 8 pounds the next. He paid for 7 pounds. For how many pounds did he still owe?

5. A man divided the 24 hours of each day equally among sleep, work, and leisure. How many hours had he for each?

6. 206,942 + 87,652 + 150,673 + 98,724 = ?

7. 546,017 − 387,524 = ?

8. $6 \times 298{,}407 = ?$

9. A newsboy started out with 194 papers and returned with 26. How many did he sell?

10. 16 bushels of oats were put into sacks. Each sack held 2 bushels. How many sacks were used?

11. There were 9 boys on each side in a baseball game. How many boys were in the game?

12. A tailor made 9 coat sleeves. For how many coats did he have the sleeves?

13. 5 boxes of figs weighed 9 ounces each. How many ounces did the 5 boxes weigh?

14. Each lamp post on Chestnut Street carries a cluster of 4 electric lights. How many electric lights are on 9 posts?

15. A milkman sold cream to five families on the same street. One of them took 7 pints a week; another 9 pints a week; another 12 pints a week; a fourth 8 pints a week, and the fifth 14 pints a week. How many pints a week did all the families take?

16. George had 17 baseballs to put into 2 boxes. The balls were to be equally divided between the two boxes. What is the largest number that could be put into each box?

17. A man paid a boy 25 cents to water his lawn. The boy watered the lawn 6 times. How many cents did he receive?

18. What did 9 apples cost at 4 cents each?

LESSON THIRTY-THREE

"SEVEN TIMES" AND "TIMES SEVEN"

I. EXPLANATION. Get from the multiplication table:

$7 \times 1 = 7$	$1 \times 7 = 7$
$7 \times 2 = 14$	$2 \times 7 = 14$
$7 \times 3 = 21$	$3 \times 7 = 21$
$7 \times 4 = 28$	$4 \times 7 = 28$
$7 \times 5 = 35$	$5 \times 7 = 35$
$7 \times 6 = 42$	$6 \times 7 = 42$
$7 \times 7 = 49$	$7 \times 7 = 49$
$7 \times 8 = 56$	$8 \times 7 = 56$
$7 \times 9 = 63$	$9 \times 7 = 63$
$7 \times 10 = 70$	$10 \times 7 = 70$
$7 \times 11 = 77$	$11 \times 7 = 77$
$7 \times 12 = 84$	$12 \times 7 = 84$

The part of the table contained in the rectangle has been studied before.

Notice that $7 \times 2 = 2 \times 7$, etc.

Learn the products in order from 7 to 84.

Drill on factor and product, as follows:

$7 \times ? = 28$, or $\frac{1}{7}$ of 8 = ?

$? \times 7 = 28$, or $7)\underline{28}$

?

II. DRILL on these combinations, as well as on those already learned. When these are known, remove them from the blackboard.

REVIEW DRILLS

1. Tables of capacity and weight.
2. Multiplication table to 7's.

UNCLASSIFIED PROBLEMS

1. 52,675 + 38,462 + 72,845 + 85,674 = ?
2. 500,186 − 294,729 = ?
3. 7 × 284,637 = ?
4. 6 cows cost 24 dollars each. How much did they all cost?
5. How many times can I feed my horses from 36 quarts of oats, if they are fed 4 quarts each time?
6. A father gave 17 cents to be divided equally between his 2 sons. How much did each receive, and how much was over?
7. If a cask holds 157 pints of vinegar, how many pints are in 5 such casks?
8. How many barrels of pickles can be bought for 28 dollars, at 4 dollars a barrel?
9. 21 birds were to be placed in 2 cages, so that the same number would be in each cage. What is the largest number of birds that could be put into each cage, and how many would be over?

10. How many envelopes 9 inches long can be placed end to end along a yardstick, if a yardstick is 36 inches long?

11. A boy studied 97 minutes each day for 5 days. How many minutes did he study altogether in that time?

12. A man earned 15 dollars one week, 17 dollars another, and 21 another. During the three weeks he spent 27 dollars. How much did he save?

13. A bushel of bird seed contains 32 quarts. If it is put up in 8 equal packages, how many quarts are there in each?

14. A boy had a bag of chestnuts. After eating 84 of them, he found by counting that 78 chestnuts were left in the bag. How many did it contain at first?

15. Just before Christmas five boys opened their savings banks. One found 98 cents; one 73 cents; one 105 cents; one 84 cents, and the fifth 27 cents. How many cents were in all the banks?

16. Harold bought 14 quarts of berries at 7 cents a quart. What did he pay for them?

17. A farmer bought 176 sheep. He then had 504. How many had he at first?

18. At a certain time of day a man 6 feet tall makes a shadow 18 feet long. The shadow is how many times as long as the man?

19. Find the cost of 7 books at 3 dollars each?

LESSON THIRTY-FOUR

INEXACT DIVISION BY 3

I. EXPLANATION. *Problem.* I have 16 apples to be placed on plates, 3 on each plate. How many plates will be used?

Imagined counting gives, $3)\underline{16.}$

Placing the pointer at 3 in the top row of the table, move it downward until 15 is reached. Notice that the next number is 18, and that 16 is not in that column at all.

"15 and how many make 16?"

"One."

Write 1 as an exponent of 16, thus, $3)\underline{16^{1}}$

Now move the pointer to the left until $5\times$ is reached. Write this as quoted thus, $3)\underline{16^{1}}$

$5\times$

5 plates will be used, and one apple will be over.

II. DRILL. Find:

$\frac{1}{3}$ of 4	$\frac{1}{3}$ of 19	$\frac{1}{3}$ of 29	$3)\underline{23}$
$\frac{1}{3}$ of 37	$\frac{1}{3}$ of 34	$\frac{1}{3}$ of 13	$3)\underline{19}$
$\frac{1}{3}$ of 5	$\frac{1}{3}$ of 8	$\frac{1}{3}$ of 28	
$\frac{1}{3}$ of 38	$\frac{1}{3}$ of 32	$\frac{1}{3}$ of 14	$3)\underline{22}$
$\frac{1}{3}$ of 7	$\frac{1}{3}$ of 10	$\frac{1}{3}$ of 26	$3)\underline{20}$
$\frac{1}{3}$ of 35	$\frac{1}{3}$ of 31	$\frac{1}{3}$ of 16	
$\frac{1}{3}$ of 17	$\frac{1}{3}$ of 11	$\frac{1}{3}$ of 25	
$\frac{1}{3}$ of 40	$\frac{1}{3}$ of 29	$\frac{1}{3}$ of 17	

REVIEW DRILLS

1. Multiplication to 7's.
2. Tables of capacity and weight.

UNCLASSIFIED PROBLEMS

1. $7 \times 483{,}972 = ?$
2. $568{,}314 - 276{,}815 = ?$
3. $43{,}608 + 58{,}712 + 143{,}285 + 97{,}605 = ?$
4. 15 oxen will make how many teams of 2 oxen each?
5. A baby took 48 ounces of milk in a day. His bottle was filled 6 times. How many ounces of milk did the bottle hold?
6. 19 marbles were to be divided between 2 boys so that each had the same number. How many marbles did each boy receive?
7. A boy, who could walk 3 miles an hour, had to take a journey of 29 miles. How many whole hours could he walk without going further than his journey's end? At the end of that time how much of his journey would be unfinished?
8. A man was in his study 12 hours one week, 17 another, and 20 another. While he was there he read all of the time except 9 hours. How many hours did he read?
9. 18 inches of ribbon were used to make 3 badges of the same size. How many inches were used for each badge?

10. Mr. Woodson left home to be gone 207 days. He has been away 159 days. How soon should he return?

11. John had five books. In one of them were 21 pictures; in another 13 pictures; in another 22 pictures; in the fourth 19 pictures, and in the fifth 14 pictures. How many pictures were in the five books?

12. 7 lots are each 32 feet wide. How many feet long must the sidewalk be in front of them?

13. Joseph hatched in an incubator 68 Plymouth Rock chickens and 47 Shanghais. 39 of these died while young. How many chickens did he raise?

14. A hall 15 feet long was covered with rugs 5 feet long. How many rugs were used?

15. 13 horses were to be made up into 3 equal teams, each team having the largest possible number of horses. How many horses were in a team, and how many were over?

16. A roll of carpet containing 48 yards was cut into strips 6 yards long. How many strips of carpet were made?

17. A certain school had 7 rooms with 41 pupils in each room. How many pupils in the school?

18. 36 ounces of tea were put up in 4-ounce packages. How many packages were made?

19. A man rowed 12 miles in 6 hours. How far did he row each hour, if his speed was the same all the time?

LESSON THIRTY-FIVE

TO READ AND WRITE NUMBERS OF THREE PERIODS

I. EXPLANATION. Numbers are composed of periods of three figures each.

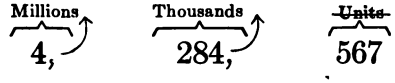
Each of these periods has a name.

The name of the first or right hand period is "units."

The name of the second period is "thousands."

The name of the third period is "millions."

In practice the periods are separated by commas, but the learner should use a diagram at first, thus,



This number is read: 4 millions, 284 thousands, 567.

Notice that each period is read as if it stood alone, with the name of the period added. This is indicated by the arrows in the diagram. The name of the first period is not read. This is indicated by the line drawn through the word "units."

Every period of a number, except the left hand or highest one, must contain three figures. If it has only two figures, a naught must be inserted at the left. If it has only one figure, two naughts must be inserted.

II. DRILL.

Millions	Thousands	~~Units~~
7,	056,	019
29,	007,	000
406,	000,	001
96,	053,	084
4,	197,	002
40,	200,	060

Read the above numbers and account for the naughts.

Pupils should learn,

First, to read numbers in the diagram.

Second, to account for the naughts.

Third, to read numbers not in the diagram.

Fourth, to write numbers in the diagram.

Fifth, to write numbers not in the diagram.

REVIEW DRILLS

1. Multiplication to 7's.
2. Inexact division by 2 and 3.
3. Read and write numbers to three periods.

UNCLASSIFIED PROBLEMS

1. Albert spent 20 cents for sacks of popcorn, at 4 cents a sack. How many sacks did he buy?

2. 4 bowls contain 78 nuts each. How many nuts in all the bowls?

3. 36,784 + 200,906 + 516,287 + 49,006 + 107,294 = ?

4. A ball of twine was at first 826 inches long. It is now 249 inches long. How many inches have been used?

5. 9 dolls were to be divided between 2 girls, so that each had the same number. What was the largest number each girl could receive?

6. 200,506 − 98,294 = ?

7. In furnishing their house, the Wilsons bought 7 pictures at 17 dollars each. What did their pictures cost?

8. 48 pints are how many gallons, if there are 8 pints in a gallon?

9. 7 × 359,078 = ?

10. 17 apples were to be placed on 3 plates, so that the same number would be on each plate. What was the largest number of apples that could be placed on each plate?

11. 7 months of the year have each 31 days. How many days in all of these seven months?

12. 11 boys were to be placed in 2 divisions of the fire squad. If the divisions were equal in number, how large a number of boys could be placed in each, and how many were over?

13. John saved 154 cents to buy a baseball. If the ball cost 129 cents, how much money had he left?

14. A street car conductor collected 45 fares on one trip, 37 on another, 26 on another, 52 on another, and 39 on the fifth. How many fares did he collect in the five trips?

LESSON THIRTY-SIX

HOW TO STUDY TIME MEASURE

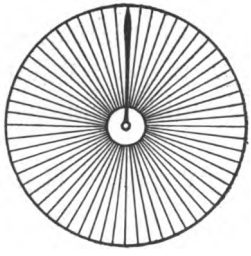
I. EXPLANATION. (a) Teach the Roman numerals to XII.

(b) Place before the pupils a clock face with only the minute hand on it.

Show (1) how far the hand moves in a minute; (2) how far it moves in an hour.

"How many minutes in an hour?"

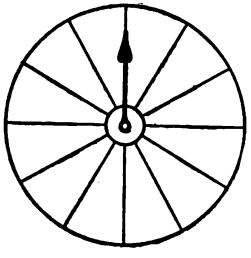
Find by counting, and write on the board, 60 minutes = 1 hour.



(Clock-face with only minutes marked on it, and only the minute hand.)

The minute hand moves around the face every hour.

Therefore, 60 minutes = 1 hour.



(Clock-face with only hours marked on it, and only the hour hand.)

The hour hand moves around the face twice every day (day and night).

Therefore, 24 hours = 1 day.

(c) Place the hour hand on the face.

Show how far it moves in an hour.

In a day (meaning a day and a night), it goes round the face twice. Then how many hours in a day?

Count and write on board, 24 hours = 1 day.

(d) Name the days of the week. Write their names on the blackboard and count them. Then write, 7 days = 1 week.

(e) Name the months of the year. Write their names on the blackboard and count them. Then write, 12 months = 1 year.

(f) After the name of each month write the number of days it contains.

Examine the list and write, 28, 30, or 31 days = 1 month.

Add the days after the names of months and write, 365 days = 1 year.

Months of Year:

January	31
February	28
March	31
April	30
May	31
June	30
July	31
August	31
September	30
October	31
November	30
December	31
	365

A PAGE FROM THE CALENDAR

1918			DECEMBER			1918
SUN	MON	TUES	WED	THURS	FRI	SAT
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	..	..	..	..

AN OLD RHYME

Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Excepting February alone,
Which hath but twenty-eight in fine,
Till leap-year gives it twenty-nine.

II. DRILL. Moving the large hand of the clock first over a minute space, then over the whole circumference, the pupil says,

"60 minutes = 1 hour."

Again moving the hour hand over an hour space, and then twice around the face, he says,

"24 hours = 1 day."

Reciting the names of the days of the week, while counting on his fingers, he says,

"7 days = 1 week."

Naming the months and counting their names, he says,

"12 months = 1 year."

Reciting the number of days in each month, he says, "365 days = 1 year."

60 minutes (min.)	=	1 hour (hr.)
24 hr.	=	1 day (da.)
7 da.	=	1 week (wk.)
28; 30; or 31 da.	=	1 month (mo.)
12 mo.	=	1 year (yr.)
365 da.	=	1 yr.

REVIEW DRILLS

1. Multiplication table.
2. Read and write numbers of three periods.
3. Time measure, liquid measure, weight, dry measure.

UNCLASSIFIED PROBLEMS

1. 463,874 + 2,708,006 + 15,080,002 + 57,894 = ?
2. 45,000,037 − 30,684,184 = ?
3. 6 × 15,432,897 = ?
4. How many leaves of a book does it take to make 18 pages?
5. A train ran from Antioch to Denton in five hours. During the first hour it ran 22 miles; during the second 24 miles; during the third 28 miles; during the fourth 25 miles; during the fifth 21 miles. How far is it from Antioch to Denton?
6. At a banquet 4 cans of ice cream were used. If each can contained 72 pints, how many pints of ice cream were used at the banquet?

7. A lady bought for Christmas 15 yards of blue ribbon and 24 yards of red ribbon. She used 32 yards of both kinds. How much ribbon had she left?

8. A lady made 20 cakes for her little girl's party. Each child at the party was given 3 cakes, and 2 cakes were left over. How many children were at the party?

9. 20 pecks of apples were placed in 5 sacks each holding a bushel. How many pecks are in a bushel?

10. A dressmaker divided 18 hours equally on 3 dresses. How long was that on each dress?

11. In a potato cellar were five bins. One held 29 bushels of potatoes; one 32 bushels; one 17 bushels; one 27 bushels, and one 19 bushels. How many bushels of potatoes were in the cellar?

12. A grocer sold 8-ounce packages of tea to each customer until he sold 56 ounces. How many customers had been served?

13. In an office 57 envelopes were used every day for 6 days. How many were used altogether?

14. A vinegar factory made 829 gallons of vinegar, and sold 784 gallons. How many gallons remained on hand?

15. 4 coats of the same value were sold for 32 dollars. How much was that apiece?

16. A grocer bought five sacks of coffee. One contained 46 pounds; another 53 pounds; another 42 pounds; another 51 pounds, and another 50 pounds. How many pounds of coffee did he buy?

LESSON THIRTY-SEVEN

"EIGHT TIMES" AND "TIMES EIGHT"

I. EXPLANATION. Get from the multiplication table:

$8 \times 1 = 8$	$1 \times 8 = 8$
$8 \times 2 = 16$	$2 \times 8 = 16$
$8 \times 3 = 24$	$3 \times 8 = 24$
$8 \times 4 = 32$	$4 \times 8 = 32$
$8 \times 5 = 40$	$5 \times 8 = 40$
$8 \times 6 = 48$	$6 \times 8 = 48$
$8 \times 7 = 56$	$7 \times 8 = 56$
$8 \times 8 = 64$	$8 \times 8 = 64$
$8 \times 9 = 72$	$9 \times 8 = 72$
$8 \times 10 = 80$	$10 \times 8 = 80$
$8 \times 11 = 88$	$11 \times 8 = 88$
$8 \times 12 = 96$	$12 \times 8 = 96$

That portion of the table enclosed in a rectangle has been studied before.

Notice that $8 \times 2 = 2 \times 8$, etc.

Learn the products in order from 8 to 96.

Drill on factor and product, as follows:

$8 \times ? = 24$, or $\frac{1}{8}$ of 24 = ?

$? \times 8 = 24$, or $8)\overline{24}$

?

II. DRILL. The devices for drill have already been given. Continue their use until the combinations are known. Then erase those combinations from the rectangular table on the blackboard.

REVIEW DRILLS

1. Multiplication table.
2. Reading and writing numbers.

UNCLASSIFIED PROBLEMS

1. $8 \times 34{,}079{,}568 = ?$
2. $278{,}436 + 1{,}269{,}879 + 14{,}374{,}965 = ?$
3. $500{,}275 - 198{,}082 = ?$
4. 16 flowers were to be planted in 3 beds, so that there should be the same number in each bed. What is the largest number that could be planted in each bed, and how many would be left over?
5. 20 bicycle wheels will be enough for how many bicycles?
6. Every summer Mary goes to the sea shore for 9 weeks. In 4 years how many weeks did she spend at the sea shore?
7. The footpath from Smith's house through the woods to the creek is 1000 yards long. Willie Smith walked from the house, 842 yards along this path. How far was he from the creek?
8. 42 oranges were sold in sacks containing 6 oranges each. How many sacks of oranges were sold?

9. On a delivery wagon were 29 pounds of butter, 75 pounds of ham, 32 pounds of sugar, 18 pounds of coffee, and other articles weighing altogether 187 pounds. How many pounds did the load weigh?

10. It takes a tailor 4 days to make a suit of clothes. How many suits can be made in 32 days?

11. 20 cows were to be placed under 3 sheds so that there should be the same number under each. How large a number could be placed under each shed, and how many would remain outside?

12. Henry sold some puppies for 3 dollars apiece, and received 15 dollars for them. How many did he sell?

13. It is 1012 miles from Allison to Bellview. A traveler started to make the trip from Allison, but stopped at Smithville, 624 miles from Allison. How far was he from Bellview?

14. A merchant bought 8 barrels of flour, each weighing 196 pounds. What did they all weigh?

15. John goes after the cows across three fields. It is 864 feet across the first field, 657 across the second, and 746 across the third. How many feet is it across the three fields?

16. 70 bushels of wheat were raised on 7 acres of land. How many bushels was that for each acre, if the same number of bushels grew on all the acres?

17. A company of soldiers contains 84 men. How many men will there be in 7 companies?

LESSON THIRTY-EIGHT

HOW TO SOLVE A CONCRETE PROBLEM REQUIRING AN OPERATION IN ADDITION OR SUBTRACTION, AND ANOTHER OPERATION IN MULTIPLICATION

I. EXPLANATION. *Problem.* A tailor in making a suit of clothes, put 5 buttons on the vest, 6 buttons on the coat, and 12 buttons on the trousers. How many buttons did he use in making 5 suits?

"How many problems?"

"Two."

"Read the first."

"A tailor in making a suit of clothes, put 5 buttons on the vest, 6 buttons on the coat, and 12 buttons on the trousers. *How many did he put on the whole suit?*"

Write it,

$$\begin{array}{r} 5 \\ 6 \\ +12 \\ \hline 21 \end{array}$$

"Read the second problem."

"A tailor used 21 buttons in making a suit of clothes. *How many buttons did he use in making 5 suits?*"

Write it,

$$\begin{array}{r} 21 \\ 5\times \\ \hline 105 \end{array}$$

REVIEW DRILLS

1. Multiplication table.
2. Tables of time, weight, and capacity.

UNCLASSIFIED PROBLEMS

1. Mrs. Brown has made 4 trips to Europe. On each trip she has stayed 5 months in London, 4 months in Paris, and 4 months in Naples. How much time has she stayed altogether in these three cities?

2. In my desk are 256 small white envelopes, 97 large white envelopes, and 175 yellow envelopes. How many envelopes are in my desk?

3. I had 15 dollars to spend for books at 2 dollars each. How many books could I buy, and how many dollars over?

4. A candy store had 129 pints of peanuts in the morning, and only 38 pints at night. How many pints were sold during the day?

5. Every day Mary spent 45 minutes in housework and 80 minutes in sewing. How many minutes did she work in 8 days?

6. John had 21 cents and gave $\frac{1}{3}$ of them to his brother. How many cents did his brother receive?

7. A dealer had 7 kegs of root beer. They had contained in the first place 28 gallons apiece, but 12 gallons had been drawn from each. How many gallons were left in them all?

8. 4 boys had altogether 28 chestnuts. If they had the same number apiece, how many had each?

9. A merchant bought a barrel of sugar containing 200 pounds. He sold 73 pounds. How many pounds remained?

10. 42 people rode past our house in automobiles, 6 in each car. How many cars went by?

11. Henry had 35 cents; he earned 49 cents; and his father gave him 25 cents. If he bought a dog for 75 cents, how many cents had he left?

12. 29 cows were to be placed in 3 fields, so that the same number would be in each. What is the largest number of cows that could be placed in each field, and how many would be over?

13. A merchant bought 7 bolts of cloth. If there were 42 yards in each bolt, how many yards did he buy?

14. A boy picked 56 bushels of apples one day, 97 the next, and 78 the third. If 24 bushels were too small to box, how many bushels of good apples did he pick?

15. If 3 feet make a yard, how many yards long are 17 feet of ribbon?

16. In a grocery store were three large cans holding kerosene. One held 63 pints, another 47 pints, and the third 126 pints. How many pints of oil were in all the cans?

17. A railroad train traveled 37 miles an hour for 6 hours. How far did it go?

LESSON THIRTY-NINE

"NINE TIMES" AND "TIMES NINE"

I. EXPLANATION. Get from the multiplication table:

$9 \times 1 = 9$	$1 \times 9 = 9$
$9 \times 2 = 18$	$2 \times 9 = 18$
$9 \times 3 = 27$	$3 \times 9 = 27$
$9 \times 4 = 36$	$4 \times 9 = 36$
$9 \times 5 = 45$	$5 \times 9 = 45$
$9 \times 6 = 54$	$6 \times 9 = 54$
$9 \times 7 = 63$	$7 \times 9 = 63$
$9 \times 8 = 72$	$8 \times 9 = 72$
$9 \times 9 = 81$	$9 \times 9 = 81$
$9 \times 10 = 90$	$10 \times 9 = 90$
$9 \times 11 = 99$	$11 \times 9 = 99$
$9 \times 12 = 108$	$12 \times 9 = 108$

The part of the table enclosed in a rectangle has already been learned.

Notice that $9 \times 2 = 2 \times 9$, etc.

Learn the products in order from 9 to 108.

Drill on factor and product, as follows:

$9 \times ? = 54$, or $\frac{1}{9}$ of $54 = ?$

$? \times 9 = 54$, $9)\overline{54}$

$?$

II. DRILL. Use the drills heretofore described until the combinations are known.

REVIEW DRILLS

1. Multiplication table.
2. Read and write numbers.

UNCLASSIFIED PROBLEMS

1. $9 \times 25{,}604{,}783 = ?$
2. A painter painted a house in 4 days. How many such houses could he paint in 28 days?
3. In making a trip to China and Japan, a steamship was gone 84 days. How many days would be spent in making 9 such trips?
4. 28 gallons of maple syrup filled 4 casks of the same size. How many gallons were in each cask?
5. $703{,}654 - 287{,}877 = ?$
6. 24 eggs were used in cakes, 4 eggs to a cake. How many cakes were made?
7. A furniture factory made 175 chairs every day for 4 days. How many chairs did it make in the 4 days?
8. 4 packages of stove polish weighed 12 ounces. How many ounces did each weigh?
9. $44{,}356{,}975 + 1{,}003{,}078 + 194{,}286{,}795 = ?$
10. At 5 dollars each, how many chains can be bought for 30 dollars?

11. James agreed to work in his garden a certain number of hours during vacation. He has already worked 35 hours, and has 49 hours still to work. How many hours did he agree to work in his garden?

12. I had 10 books to place on 2 shelves, putting the same number on each shelf. How many books could I place on each shelf?

13. John worked 212 minutes, and Charles 197 minutes. How many minutes did John work longer than Charles?

14. If a boy rides on his bicycle 624 hours a year, how many hours will he ride in 7 years?

15. It took a gardener 2 days to spade a large flower bed. How many such flower beds can he spade in 15 days?

16. A forest contained 1975 fir trees, 378 maple trees, and 927 cedar trees. A fire destroyed 1479 trees. How many trees were left in the forest?

17. A lady had five boxes of writing paper. In one box were 29 sheets of paper; in another box 17 sheets; in a third box 23 sheets; in a fourth box 15 sheets, and in the fifth box 27 sheets. How many sheets of paper had she?

18. The bean poles in a garden were each 7 feet high. Their whole length was 56 feet. How many bean poles were in the garden?

19. 54 dollars was received for hats worth 6 dollars each. How many were sold?

LESSON FORTY

HOW TO MULTIPLY WITH MORE THAN ONE FIGURE IN THE MULTIPLIER

I. EXPLANATION. *Problem.* Find 36×857.

Write it thus,

$$\begin{array}{r} 857 \\ 36\times \\ \hline 5142 \\ 2571 \\ \hline 30852 \end{array}$$

$6 \times 7 = 42$. Put down 2 *under 6*, and carry 4.

$6 \times 5 = 30$, and the 4 carried makes 34. Put down 4, and carry 3.

$6 \times 8 = 48$, and the 3 carried makes 51. Put down both figures.

$3 \times 7 = 21$. Put down 1 *under 3*, and carry 2.

$3 \times 5 = 15$, and 2 carried makes 17. Put down 7, and carry 1.

$3 \times 8 = 24$, and 1 carried makes 25. Put down both figures.

Add the products.

Notice that we begin to write each product under the figure of the multiplier which produces it.

II. DRILL.

$$\begin{array}{r} 3456 \\ 204\times \\ \hline 13824 \\ 6912 \\ \hline 705024 \end{array}$$

Observe that the first partial product is produced by 4 units, and we begin to write it under units.

The second partial product is produced by 2 hundred, and we begin to write it under hundreds.

$$\begin{array}{r} 5976 \\ 450\ \times \\ \hline \end{array}$$

How many partial products?

Where do we begin to write the first partial product? Why? Where do we begin to write the second? Why?

After we add these two partial products, we write a naught in the units' place.

REVIEW DRILLS

1. Multiplication table.
2. Inexact division by 2 and 3.

UNCLASSIFIED PROBLEMS

1. $37 \times 2567 = ?$

2. In a grocery store were three barrels containing sugar. One held 87 pounds, another 156 pounds, and the third 98 pounds. The merchant sold to customers 214 pounds of sugar. How many pounds had he left?

3. A bucket holds 3 gallons of water. How many times can it be filled from a barrel containing 23 gallons?

4. Mr. Wilson had 8674 dollars in the bank. He spent 3415 dollars for a house, 876 dollars for furniture, and 129 dollars for improvements. How many dollars remained in the bank?

5. A teacher divided 40 stars among 8 pupils, giving each the same number. How many stars did each pupil receive?

6. $204 \times 3916 = ?$

7. A tailor charged 7 dollars for making a coat. How many coats must he make to earn 42 dollars?

8. A baker bought 37 sacks of flour from one mill, 86 sacks from another, and 52 sacks from another. He used 112 sacks. How many sacks had he on hand?

9. George worked in a store 27 days. Afterwards he ran errands 43 days. William worked in a factory 53 days. How many days did George work more than William?

10. 30 pounds of sugar were put into 6 sacks, so that each sack had the same quantity of sugar. How many pounds were in each sack?

11. $90 \times 7643 = ?$

12. At 5 dollars each, how many chairs can be bought for 25 dollars?

13. After working some time in a store, Mr. Williams was a street car conductor for 17 weeks. If he worked 41 weeks in both places, how long did he work in the store?

14. $4080 \times 167{,}249 = ?$

LESSON FORTY-ONE

INEXACT DIVISION

I. EXPLANATION. Find $\frac{1}{4}$ of 22.

This means, $4 \times ? = 22$

$4 \times 5 = 20$

$4 \times 6 = 24$

22 does not occur in the table, so we use the next lower number, 20. $\frac{1}{4}$ of 20 = 5. Therefore $\frac{1}{4}$ of 22 is 5, and 2 over.

This is written $\frac{1}{4}$ of $22^2 = 5$.

II. DRILL. Solve:

4)19	$\frac{1}{5}$ of 21 =	6)45
4)23	$\frac{1}{5}$ of 37 =	6)49
4)27	$\frac{1}{5}$ of 42 =	6)39
4)35	$\frac{1}{5}$ of 47 =	6)41
4)43		

REVIEW DRILLS

1. Tables of weight, capacity, and time.
2. Multiplication table.

UNCLASSIFIED PROBLEMS

1. $2500 \times 1{,}368{,}274 = ?$

2. It took a teacher 4 hours to mark a set of examination papers. How many sets of papers could she mark in 18 hours?

3. A farmer had 225 bushels of potatoes in one cellar, and 352 in another. If he sold 469 bushels, how many bushels had he left?

4. April has 30 days. How many weeks of 7 days each are in this month, and how many days are over?

5. If there are 24 quarts of ink in each of 6 kegs, how many quarts will be left after 95 quarts are used?

6. At a church fair 314 ounces of candy were sold on the first night; 297 ounces on the second night; and 323 ounces on the third night. How many ounces of candy were sold in the three nights?

7. A lady baked 22 cookies, and gave 4 apiece to some children. How many children could she serve?

8. Harold had 97 cents in his bank and 27 cents in his pocket, and his mother gave him 12 cents for cleaning the yard. If he pays his sister 40 cents which he owes her, how many cents will he have left?

9. A willow stick 18 inches long was made into 4 whistles of the same length. How long was each whistle, if they were made as long as possible; and how much of the stick was not used?

10. From Millville to Gaston is 38 miles; from Gaston to Ridgely 14 miles, and from Ridgely to Harmony 27 miles. How many miles is it from Millville to Harmony?

11. $805 \times 400{,}654 = ?$

12. School is closed 12 weeks each summer. A boy spends every year 8 weeks of this vacation at home, and the remainder in the mountains. How many weeks does he spend in the mountains in 5 years?

13. If 2 yards of cloth are used in making a coat, how many such coats can be made of 17 yards?

14. 27 fans were to be divided equally among 5 girls. What is the largest number of fans each girl could receive, and how many would be left?

15. $264 \times 565{,}742 = ?$

16. Mr. Brown lives on a lot 187 feet wide. Mr. Smith's lot is 68 feet narrower than Mr. Brown's. How wide is Mr. Smith's lot?

17. Henry used 45 minutes in preparing his arithmetic, and 37 minutes in preparing his geography. But 13 minutes of this time were used in getting out and putting away his books and paper. How many minutes did he spend in actual study?

18. 24 ounces of pepper were put into packages of 4 ounces each. How many packages did it make?

19. 27 houses were sold for 1500 dollars apiece. What was received for them?

LESSON FORTY-TWO

"TEN TIMES" AND "TIMES TEN"

I. EXPLANATION. Get from the multiplication table:

$10 \times 1 = 10$	$1 \times 10 = 10$
$10 \times 2 = 20$	$2 \times 10 = 20$
$10 \times 3 = 30$	$3 \times 10 = 30$
$10 \times 4 = 40$	$4 \times 10 = 40$
$10 \times 5 = 50$	$5 \times 10 = 50$
$10 \times 6 = 60$	$6 \times 10 = 60$
$10 \times 7 = 70$	$7 \times 10 = 70$
$10 \times 8 = 80$	$8 \times 10 = 80$
$10 \times 9 = 90$	$9 \times 10 = 90$
$10 \times 10 = 100$	$10 \times 10 = 100$
$10 \times 11 = 110$	$11 \times 10 = 110$
$10 \times 12 = 120$	$12 \times 10 = 120$

The part of the table enclosed in the rectangle has been studied before.

Notice that $10 \times 2 = 2 \times 10$, etc.

Learn to say the products in order from 10 to 120.

Drill on factor and product, as follows:

$10 \times ? = 70$, or $\frac{1}{10}$ of $70 = ?$

$? \times 10 = 70$, or $10)\overline{70}$

$?$

II. DRILL daily in the ways already described until the combinations are known.

REVIEW DRILLS

1. Inexact division by 4 and 5.
2. Multiplication table.

UNCLASSIFIED PROBLEMS

1. If 347 automobiles cross Market Street bridge every day, how many automobiles cross the bridge in 6 days?

2. 563,784 + 2,070,568 + 3,597 + 486,594 = ?

3. Mary had 49 cents in her bank with which she wished to buy 5 paper dolls at the same price. What was the highest price she could pay, and how many cents had she left?

4. In a regiment of cavalry there are 1008 men. In a regiment of infantry there are 840 men. How many more men are in a regiment of cavalry than in a regiment of infantry?

5. 21 chair legs will be enough for how many common chairs?

6. A peanut vender sold 21 pecks of peanuts in 3 days. How many pecks was that a day?

7. 83,070 × 564,965 = ?

8. A bottle holds 3 pints of ink. How many such bottles can be filled with 29 pints?

9. A school of 19 rooms had 42 desks in each of the rooms. If 176 desks had broken hinges, how many were fit for use?

10. 15 pints of vinegar leaked from a bucket. There remained 42 pints. How many pints were in the bucket at first?

11. 8 boys were of the same age, and their ages added together made 96 years. How old was each boy?

12. Mary received for Christmas 3 boxes of candy. Each box contained 6 ounces of caramels, 8 ounces of chocolate creams, and 3 ounces of taffy. How many ounces of candy did she receive?

13. In a school of 8 rooms, each room contains 32 ordinary desks and 16 adjustable desks. How many school desks are in the building?

14. William has 26 postal cards to be placed on 4 pages of an album, the same number on each page. What is the largest number of cards that can be placed on each page, and how many will be left over?

15. A boy attended school 5 hours a day for 19 days. If he studied arithmetic 16 hours during that time, how many hours were left for his other studies?

16. A storekeeper had 4 barrels containing oil. One held 30 gallons, another 31 gallons, a third 29 gallons, and a fourth 24 gallons. If he sold 12 gallons from the first barrel, 13 gallons from the second, 11 gallons from the third, and 6 gallons from the fourth, how many gallons had he left in the four barrels?

17. A man had 8000 dollars and spent 6960 for a farm. How much money had he left?

LESSON FORTY-THREE

"ELEVEN TIMES" AND "TIMES ELEVEN"

Get the following from the multiplication tables:

$11 \times 1 = 11$	$1 \times 11 = 11$
$11 \times 2 = 22$	$2 \times 11 = 22$
$11 \times 3 = 33$	$3 \times 11 = 33$
$11 \times 4 = 44$	$4 \times 11 = 44$
$11 \times 5 = 55$	$5 \times 11 = 55$
$11 \times 6 = 66$	$6 \times 11 = 66$
$11 \times 7 = 77$	$7 \times 11 = 77$
$11 \times 8 = 88$	$8 \times 11 = 88$
$11 \times 9 = 99$	$9 \times 11 = 99$
$11 \times 10 = 110$	$10 \times 11 = 110$
$11 \times 11 = 121$	$11 \times 11 = 121$
$11 \times 12 = 132$	$12 \times 11 = 132$

The part of the table enclosed in the rectangle has been studied before.

Notice that $11 \times 2 = 2 \times 11$, etc.

Learn to say the products in order from 11 to 132.

Drill on factor and product, as follows:

$11 \times ? = 88$, or $\frac{1}{11}$ of $88 = ?$

$? \times 11 = 88$, or $11)\underline{88.}$

$?$

REVIEW DRILLS

1. Inexact division by 6 and 7.
2. Multiplication table.

UNCLASSIFIED PROBLEMS

1. Our box hedge is 87 yards long, and our hedge of barberry 65 yards long. If we remove 95 yards of hedge, how many yards will be left?

2. In each of 3 drawers in a desk were 14 two-cent stamps, 12 one-cent stamps, and 4 five-cent stamps. How many stamps were in the desk?

3. Harry had 25 cents with which he wished to buy 3 tops at the same price. What is the highest price he could pay for each, and how many cents would he have left?

4. $463 \times 59{,}384 = ?$

5. Mr. Robinson sold a horse for 126 dollars. He then bought a cow for 67 dollars. How many dollars remained?

6. 5 quarts of berries make a jar of jam. How many jars of jam can be made from 29 quarts of berries?

7. A horse dealer sold 168 horses. He still had 289 horses for sale. How many horses had he at first?

8. If it took 8 inches of ribbon to make each bow, how many bows did 56 inches make?

9. A man 56 years old has lived the same length of time in each of 7 states. How long has he lived in each state?

10. From Springfield to Homesville is 48 miles. From Springfield through Homesville to Holt is 55 miles. If Mr. Nelson walks from Homesville to Holt and back again 4 times a week, how far does he walk each week?

11. From a bin containing 759 bushels of oats, 468 bushels were sold. How many bushels remained?

12. Mrs. Johnson refurnished 6 rooms of her house spending on each 3 dollars for curtains, 20 dollars for carpet, and 30 dollars for furniture. What did it cost her?

13. If a girl picked 6 quarts of berries each day, how many days did it take her to pick 30 quarts?

14. In 7 trips, a street car conductor *averaged 64 cash fares and 32 transfers. How many passengers did he carry altogether?

15. Mr. Carson kept his chickens in 5 yards. He bought 30 pecks of feed. How much was that for each yard, all the yards receiving the same quantity?

16. A butcher had 5 sides of beef each weighing 175 pounds. If he sold 584 pounds of beef, how much had he left?

17. 24 horses were sold for 275 dollars each. 2500 dollars of the money received was used to pay debts. The remainder was placed in the bank. How much money was placed in the bank?

*_Averaged._ This means that the whole number of passengers carried on the 7 trips is the same as if he had carried 64 cash fares and 32 transfers each trip.

LESSON FORTY-FOUR

"TWELVE TIMES" AND "TIMES TWELVE"

Get the following from the multiplication table:

$12 \times 1 = 12$	$1 \times 12 = 12$
$12 \times 2 = 24$	$2 \times 12 = 24$
$12 \times 3 = 36$	$3 \times 12 = 36$
$12 \times 4 = 48$	$4 \times 12 = 48$
$12 \times 5 = 60$	$5 \times 12 = 60$
$12 \times 6 = 72$	$6 \times 12 = 72$
$12 \times 7 = 84$	$7 \times 12 = 84$
$12 \times 8 = 96$	$8 \times 12 = 96$
$12 \times 9 = 108$	$9 \times 12 = 108$
$12 \times 10 = 120$	$10 \times 12 = 120$
$12 \times 11 = 132$	$11 \times 12 = 132$
$12 \times 12 = 144$	$12 \times 12 = 144$

The combinations enclosed in the rectangle have already been learned.

Notice that $12 \times 2 = 2 \times 12$, etc.

Learn to say the products in order from 12 to 144.

Drill on factor and product, as follows:

$12 \times ? = 96$, or $\frac{1}{12}$ of 96 = ?

$? \times 12 = 96$, or $12)\overline{96.}$

?

REVIEW DRILLS

1. Inexact division by 8 and 9.
2. Multiplication table.

UNCLASSIFIED PROBLEMS

1. Johnson was out in camp 28 days. He fished 10 days and hunted 9 days. The remainder of the time he rested in camp. How many days did he rest?

2. If there are 9 boys on each side in a game of baseball, how many boys are engaged when 3 games are being played at the same time?

3. $4{,}086 \times 25{,}697 = ?$

4. A boy fastened 238 feet of string to his kite. A strong wind broke the string and blew the kite away. The boy had 149 feet of string remaining. How many feet of string blew away with the kite?

5. A glovemaker finished 40 fingers in a day. How many gloves did he make?

6. $\frac{1}{8}$ of $67 = ?$ $\frac{1}{5}$ of $39 = ?$ $\frac{1}{4}$ of $42 = ?$

7. 24 boys had each 35 common marbles and 12 alleys. How many marbles did they all have?

8. 12 kegs of cider contained 6 gallons each. How much cider was in all of them?

9. The time-keeper's book shows that Michael Jones has worked 72 hours. How many days has he worked, if 8 hours are a day's work?

10. Alice had 86 inches of ribbon with which to make 12 sashes for her doll. How long was each

sash, if they were of the same length, and how many inches of ribbon were left over?

11. Our gray horse cost 135 dollars. The bay one cost 98 dollars. How much more did the gray horse cost than the bay one?

12. A boat makes a trip in 5 days. In 22 days what is the largest number of trips it can make?

13. A medicine chest contained 20 ounces of quinine, 84 ounces of calomel, and 36 ounces of other drugs. How many ounces of drugs would 3 such chests contain?

14. There are 234 pupils in Garfield School, 483 in Lincoln School, and 184 in Taylor School. How many pupils are in the three schools?

15. How many maps each 12 inches long can be made from 84 inches of drawing paper?

16. A storekeeper had 5 boxes, each containing 24 quart bottles of grape juice. He sold 56 quarts. How much had he left?

17. In an army there were 36 regiments of soldiers. The regiments contained an average of 857 men. How many soldiers were in the army?

18. A sheep raiser had 10,000 sheep in four flocks. In three of the flocks there were altogether 7273 sheep. How many sheep were in the fourth flock?

19. A farmer dug 184 bushels of potatoes in one field, 257 bushels in another field, and 98 in a third field. How many bushels did he dig in all three of the fields?

20. Find the cost of 4 pianos at $467 each?

LESSON FORTY-FIVE

LINEAR MEASURE MEMORIZED

I. EXPLANATION. Place on the number table a yard stick, a foot rule, and some one-inch pieces of card board.

Make the pupils familiar with the names of each.

"How many inch pieces are as long as the foot rule?"

1 foot (ft.)

12 in. = 1 ft.

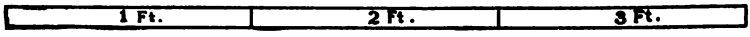
12 inches (in.)

Find by counting and write, 12 inches = 1 foot.

"How many feet equal a yard?"

Find by counting and write, 3 feet = 1 yard.

1 yard (yd.)



3 feet

3 ft. = 1 yd.

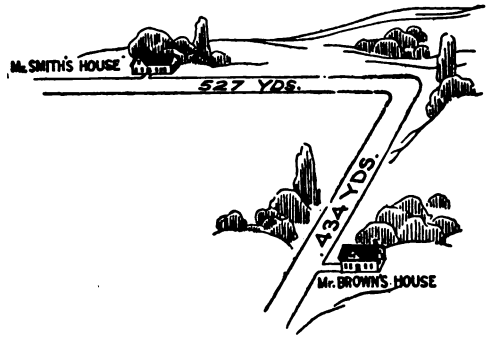
1760 yd. = 1 mile (mi.)

A mile is as far as from —— to ——. 1760 yards = 1 mile.

II. ILLUSTRATIVE PROBLEMS.

(a) Mr. Smith lived in the country. His house was near the road. 527 yards east of his house the road turned to the south. 434 yards south of the turn is Mr. Brown's house.

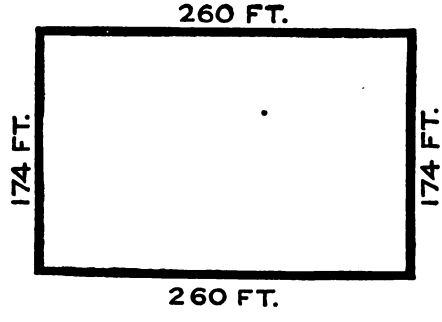
How far from Mr. Smith's house to Mr. Brown's house?



How much farther from the turn of the road is Mr. Smith's house than Mr. Brown's?

(b) A figure with four straight sides and four square corners is called a *rectangle.*

A square-cornered field is 260 feet long and 174 feet wide (see drawing).



If a boy started at the upper left corner and walked around the field, how many feet would he travel?

UNCLASSIFIED PROBLEMS

1. 15 boys were playing in the park at 3 o'clock. Between 3 and 4 o'clock 25 more boys came to play, and 6 went home. How many boys were playing in the park at 4 o'clock?

2. A lady bought for her kitchen 14 yards of linoleum. She had to cut it into strips 4 yards long. How many strips did she cut, and how many yards had she over?

3. 6 plumes for hats cost 30 dollars. How much was that apiece, if they all cost the same amount?

4. It took 4 yards of linoleum to cover the kitchen floor of a new house, and 3 yards for the bathroom. How many yards were needed for 8 such houses?

5. A boarding house bought 374 pint cans of soup stock. After using 189 cans, how many pints were on hand?

6. If 22 oranges were placed 7 in a basket, how many baskets were needed?

7. On the first of the month a bookseller had on hand 17 quarts of black ink. During the month he bought 18 quarts and sold 19. How many quarts of black ink had he at the end of the month?

8. A man built 63 feet of fence in 7 days. How many feet did he average a day?

9. A picture frame is 9 inches long and 6 inches wide. How long was the piece of moulding from which it was made? (Draw figure.)

10. At 6 cents a quart, how many quarts of currants may be bought for 38 cents?

11. A stationer bought 7 boxes of penholders, 36 in each box. After selling 198 penholders, how many had he left?

12. 38 horses are to be placed in 9 equal teams. What is the largest number of horses that can be placed in each team, and how many extra horses will there be?

13. The road on which Mr. Johnson lives runs east. 1565 yards east of his house the road turns north, and 1284 yards north turns to the east again. 906 yards east of the second turn is the Eagle tavern. How far is it along the road from Johnson's house to the Eagle tavern? (Make drawing.)

14. There are 4 pecks in a bushel. How many pecks in 108 bushels?

15. A druggist had 47 cakes of soap on one shelf, 53 cakes on another, and 34 on another. If he sold 72 cakes, how many cakes had he left?

16. 34 gallons of cider were placed in kegs holding 6 gallons each. How many kegs were filled?

17. Mr. Martin built a fence around a lot 98 feet long and 43 feet wide. How many feet long was the fence?

18. 63 pounds of coffee filled 7 equal sacks. How many pounds were in each sack?

19. How many orders for 5 pints each can be filled from a can of ice cream holding 22 pints?

CHAPTER IV

STEPS OF FOURTH YEAR

LESSON FORTY-SIX

REVIEWS AND DRILLS

I. Tables of artificial pluralities.

(a) The pupil touches each unit while naming the connecting number. Thus:

2 pints (touching pint measure) = 1 quart (touching quart measure).
4 quarts (touching quart measure) = 1 gallon (touching gallon measure).

8 quarts (touching quart measure) = 1 peck (touching peck measure).
4 pecks (touching peck measure) = 1 bushel (touching bushel measure).

16 ounces (touching ounce weight) = 1 pound (touching pound weight).
2000 pounds (touching pound weight) = 1 ton (wave arm to indicate size).

60 minutes (moving minute hand 1 space) = 1 hour (moving minute hand around face of clock).

24 hours (moving hour hand twice around face of clock) = 1 day.

7 days (naming days of week) = 1 week.

28, 30 or 31 days = 1 month.

12 months (naming months) = 1 year.

365 days = 1 year.

12 inches (touching an inch measure) = 1 foot (touching foot rule).

3 feet (touching foot rule) = 1 yard (touching yardstick).

(b) After pupils can recite tables as described above, have them recited without objective reference, except when a pupil fails to remember a connecting number. In such case he may be sent to the number table to examine the units whose ratio he has forgotten.

II. Multiplication table with exact and inexact divisions.

III. Extensions of addition table.

UNCLASSIFIED PROBLEMS

I. How many years are there in 98 months?

II. Our house tank will hold 880 gallons when full. It now contains 257 gallons. How many gallons more will be needed to fill it?

3. How far is it around a rectangle 9 inches long and 7 inches wide?

4. From 17 gallons of grape juice a man filled 7 demijohns of the same size. What is the largest number of gallons that could be put into each demijohn, and how many gallons would be left over?

5. Mrs. Gray was in Europe 24 months. She spent 8 months in Scotland, 7 months in England, and the remainder in Paris. How long was she in Paris?

6. A factory had so many orders for goods that it was kept running night and day. Each shift of workmen worked 8 hours out of 24. How many shifts were there?

7. For 9 years a lady spent 5 months of each year in Europe. If 20 months of this time were passed in Italy, how much time was spent in other parts of Europe?

8. The carpenter-work on a house cost 1278 dollars, the plumbing 217 dollars, and other work 456 dollars. How much would it cost to build 9 such houses?

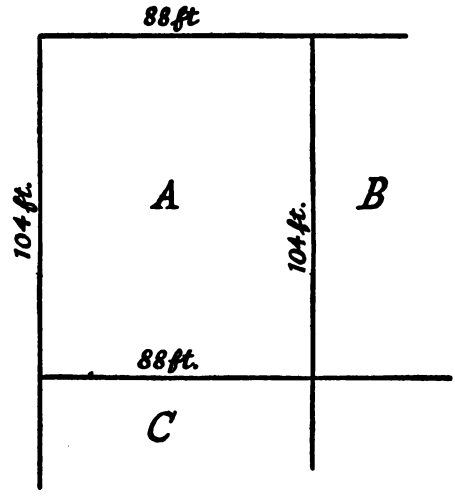
9. The 45 boys working in a factory were formed into baseball teams with 9 boys on each team. How many teams did they make?

10. 34 pounds of raisins were to be divided as nearly as possible into six equal parts. How many whole pounds were put into each part?

11. 24 ounces of candy were put on Christmas plates, 7 ounces on each plate. How many plates were filled?

12. Last Monday 23 pupils entered our school, making 217 pupils in all. How many pupils had we before the new ones entered?

13. This drawing shows Mr. A's lot, also the position of Mr. B's property east of A's, and Mr. C's south of A's.



A fence is to be built around Mr. A's property. For the fence between A and B each is to pay one-half. For the fence between A and C each is to pay for one-half. For how many feet of fence will each of the three men pay?

14. There are 5280 feet in a mile. William walked 3500 feet less than 5 miles. How many feet did he walk?

15. 108 oranges are arranged in 9 equal piles. How many oranges are in each pile?

16. Mr. Sanders sold a farm of 145 acres for 70 dollars an acre. He also sold 12 building lots at 450 dollars each. What did he receive for the farm and lots?

LESSON FORTY-SEVEN

SHORT DIVISION (PARTITION)

I. EXPLANATION. *Problem.* Find $\frac{1}{6}$ of 192.

Find $\frac{1}{6}$ of 19.

$\frac{1}{6}$ of $19^{1} = 3$.

Now annex the 2 which taken with the one over makes 12.

$\frac{1}{6}$ of $12 = 2$.

The whole solution now becomes,

$\frac{1}{6}$ of $19^{1}2 = 32$.

II. DRILL. Find:

$\frac{1}{3}$ of 192	$\frac{1}{2}$ of 38	$\frac{1}{4}$ of 72
$\frac{1}{4}$ of 176	$\frac{1}{3}$ of 204	$\frac{1}{5}$ of 90
$\frac{1}{5}$ of 215	$\frac{1}{4}$ of 184	$\frac{1}{6}$ of 124
$\frac{1}{6}$ of 138	$\frac{1}{5}$ of 175	$\frac{1}{7}$ of 203
$\frac{1}{7}$ of 224	$\frac{1}{6}$ of 264	$\frac{1}{8}$ of 232
$\frac{1}{8}$ of 176	$\frac{1}{7}$ of 238	$\frac{1}{9}$ of 261
$\frac{1}{9}$ of 288	$\frac{1}{8}$ of 184	$\frac{1}{7}$ of 196

UNCLASSIFIED PROBLEMS

1. Find $\frac{1}{2}$ of 538.

2. A door is 84 inches high and 32 inches wide. How many inches long is a cord which will just reach around the edge of the door?

3. How many cents are 234 nickels?

4. Find $\frac{1}{2}$ of 1172.

5. To cross the ocean and unload her cargo and load another takes a steamship 8 days. How many trips can she make in 56 days?

6. A stenographer can write a letter in 7 minutes. How many of the same length can she write in 63 minutes?

7. David had 32 rabbits to place in 5 hutches, so that the same number of rabbits would be in each hutch. How many rabbits did he put into a hutch?

8. An average of 215 children were playing on each of 5 city playgrounds at 1 o'clock Saturday afternoon. If 329 of them had gone home before 3 o'clock, how many were then playing on the five playgrounds?

9. If candles are sold 6 in a box, how many candles will be sold in 186 boxes?

10. An ice cream dealer had a number of orders, each for 7 pints of ice cream. How many of them could he fill if he had 58 pints on hand?

11. A field with square corners is 297 feet long and 184 feet wide. How long is the fence which encloses the field?

12. Find $\frac{1}{5}$ of 280.

13. A doctor during his office hours gives on an average 4 minutes to each patient. How many patients can he see in 52 minutes?

14. A father bought for each of his 5 sons, a hat for 2 dollars, a coat for 5 dollars, and a pair of shoes for 4 dollars. What did they all cost him?

15. To go from Arton to Denton, we must go north 1864 yards, then east 975 yards, then south 256 yards, then east 2964 yards, then north 1587 yards. How far is it from one town to the other?

LESSON FORTY-EIGHT

SHORT DIVISION (MEASUREMENT)

I. EXPLANATION. *Problem.* How many times can 6 be taken from 192?

$$6\overline{)19^{1}2}\quad 32$$

"How many times can 6 be taken from 19?"
"3 times, and 1 over."
"How many times can 6 be taken from 12?"
"Twice."
"Therefore, 6 can be taken from 192, 32 times."

II. DRILL. Solve:

3)1131	8) 672
4)2232	9) 198
5)1620	2)1356
6) 342	3)1725
7) 511	4)2556

UNCLASSIFIED PROBLEMS

1. Find $\frac{1}{4}$ of 304.

2. Willie has a strip of leather 13 feet long which he wishes to cut into 6 equal top cords, each cord to be as many feet long as possible. How long will each cord be, and how many feet of leather will be left over?

3. A rectangle is 27 inches long and 18 inches wide. How many inches around it?

4. If 2 ounces of silver are needed to plate a small tray, how many such trays will 17 ounces plate?

5. In one delivery wagon there is a load weighing 329 pounds. In a second wagon the load is 78 pounds heavier. How heavy is the load in the second wagon?

6. Find $\frac{1}{3}$ of 27 pounds.

7. A boy raised 274 chickens one year, 396 the next, and 298 the third. How many chickens did he raise in the three years?

8. At 9 dollars apiece, how many raincoats were sold for 828 dollars?

9. Mary takes 15 days to read a book from the library. How long will it take her to read 17 such books?

10. How many times can 9 be taken from 207?

11. From a board there were cut a shelf 42 inches long, a seat for a swing 37 inches long, and there was left a piece of the board 41 inches long. How many inches long was the board at first?

12. How many times can 2 be taken from 138?

13. Each of 4 bins contained 148 pecks of oats. If 29 pecks be used from each bin, how many pecks will be left in all of them?

14. In each of 8 houses there were 4 front windows, 9 side windows, and 3 rear windows. How many windows in the houses?

15. At an office the telephone calls average 32 a day. At this rate how many calls are there in 24 days?

LESSON FORTY-NINE

CONCRETE PROBLEMS IN SHORT DIVISION

I. EXPLANATION. *Problem.* There are 7 buttons on a shoe. 504 buttons will be enough for how many shoes?

"What would you do first?"

"Count 504 buttons from the supply."

"What next?"

"Count 7 at a time from the 504 until none are left."

"Write it."

Pupils write, $7\overline{)50^{1}4}$, and then say,

72

"There will be enough buttons for 72 shoes."

Problem. 441 gallons of water filled 7 barrels of the same size. How many gallons of water in each barrel?

"What would you do first?"

"Count 441 gallons from the supply."

"What next?"

"Count one at a time in 7 places till all are gone."

"Write it."

Pupils write, $\frac{1}{7}$ of $44^{2}1 = 63$.

Then they say "There will be 63 gallons in each large barrel."

REVIEW DRILLS

1. Read and write numbers of three periods.
2. Inexact division.

UNCLASSIFIED PROBLEMS

1. On Mr. Smith's fruit farm are 1250 apple trees, 1873 prune trees, and 1547 peach trees. How many fruit trees are on his farm?

2. Each shelf of a china closet held 8 large dishes and 13 small ones. If there were 6 shelves in the closet, how many dishes did it hold?

3. A spelling match was held in a room containing 38 pupils. How many pupils were on each side?

4. A man plans to build 21 stores on 4 blocks, with the same number on each block. What is the greatest number he can put on each block, and how many stores will be left unbuilt?

5. Each of 7 houses on Thomas street has 38 feet of hose for the front yard, and 23 feet for the back yard. How many feet of hose have all the houses?

6. In a stable were three feed chests. One of them contained 187 bushels of oats, another 89 bushels, and the third 97 bushels. After 124 bushels of oats have been fed to the horses, how many bushels will remain in the feed chests?

7. How many bottles each holding 3 pints can be filled with 21 pints of mucilage?

8. In a book case were five shelves. On the top shelf were 27 books; on the next 28 books; on the next 15 books; on the next 20 books, and on the bottom shelf 24 books. If 45 books should be moved from the book case, how many books would it then contain?

9. There were 272 fish in 4 barrels. How many did that average to the barrel?

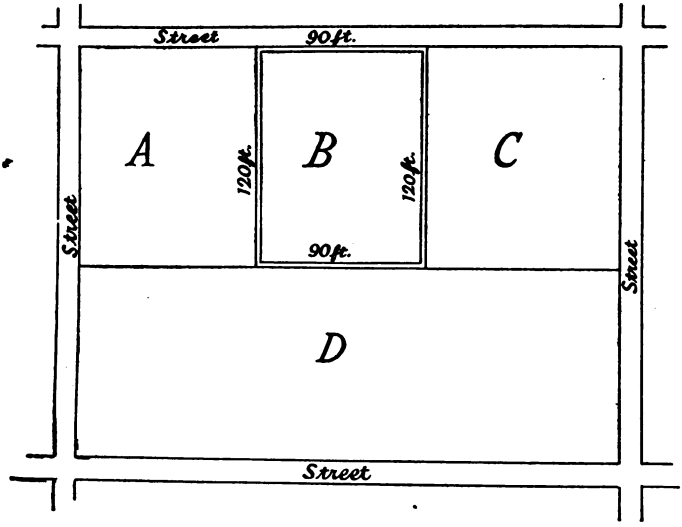
10. A storekeeper received in one day 185 dollars for hats at 5 dollars apiece. How many hats did he sell?

11. A singing teacher spends 12 minutes in each room of an 18-room school building. How much time does she spend in all the rooms?

12. At 3 cents apiece how many postage stamps can be bought for 141 cents?

13. There were 240 pints of cider in 5 casks. How many pints was that to the cask?

14. If a fence is built around B's lot, for how many feet of fence must B pay?



For how many feet of fence must A, D, and C each pay?

LESSON FIFTY

READING AND WRITING UNITED STATES MONEY

I. EXPLANATION. The money of the United States is usually written as dollars and cents.

100 cents = 1 dollar.

The dollar mark $ placed before a number shows that it means a number of dollars; thus,

$7 = 7 dollars	$100 = 100 dollars
$15 = 15 dollars	$5290 = 5290 dollars

A dot called the decimal point is written between dollars and cents. Thus,

$18.94 = 18 dollars and 94 cents.
$110.35 = 110 dollars and 35 cents.
$100.06 = 100 dollars and 6 cents.

Notice that when written in this way, cents is always expressed by two figures. In the last amount a naught was placed to the left of the 6 to express 6 cents.

Sometimes when the amount of money is less than a dollar it is expressed in this same way. In such cases nothing is written to the left of the decimal point. Thus:

$.19 = 19 cents	$.10 = 10 cents
$.46 = 46 cents	$.04 = 4 cents

II. DRILL. Read the following:

$1,012.16	$1.97
900.05	.07
257.09	.74
1,600.74	.63

Dictate additional exercises.

Write the following:

(1) 597 dollars, 13 cents.
(2) 80 dollars, 20 cents.
(3) 412 dollars, 3 cents.
(4) 97 cents.
(5) 8 cents.
(6) 2 dollars, 4 cents.

Dictate additional exercises.

REVIEW DRILLS

Give a short drill every day in writing amounts of money. After they are written they should be read aloud.

UNCLASSIFIED PROBLEMS

1. During the months of March, April, and May, Mr. Flint's hens laid 814 eggs. He sold 546 of them. How many did he keep for his own use?

2. Mary had 45 minutes in which to write letters. If it took 9 minutes for each letter, how many did she write?

3. What is the distance around a rectangle 14 feet long and 9 feet wide?

4. How many sacks each holding 6 pecks will be needed for 1284 pecks of potatoes?

5. In a bookcase containing 6 shelves, there were 19 books on each shelf. If 47 books were removed from the bookcase, how many remained?

6. A large bowl contained 21 walnuts and 38 almonds. How many nuts altogether would 7 such bowls contain, if they were all filled in the same way?

7. A boy desired to put 37 marbles into 5 bags, so that there would be the same number in each bag. What is the largest number he could put into each bag, and how many would be over?

8. A farmer had 326 bushels of turnips to feed his hogs. How long would the turnips last, if he fed the hogs 4 bushels a day?

9. 7 months of the year have each 31 days. How many days in all of these months?

10. A dealer bought 764 sheep on Monday, 97 sheep on Tuesday, and 697 sheep on Wednesday. How many sheep did he buy in the three days?

11. A city had 2 reservoirs, each containing 38,000,000 gallons of water. If 4,289,794 gallons leaked from them, how many gallons were left?

12. A lady had 36 pieces of candy which she put into sacks for some children. If she put 4 pieces into each sack, to how many children could she give a sack apiece?

13. 59 boys were to be divided into 3 equal companies for drill. How large a number of boys could be placed in each company and how many would be left?

14. A thin board 17 feet long is to be cut into pieces 2 feet long, each piece to be used for the cover of a box. How many boxes can be covered and how many feet of board will be left?

15. How many times can 4 be taken from 340?

LESSON FIFTY-ONE

ADDING AND SUBTRACTING UNITED STATES MONEY

I. EXPLANATION. (a) \$593.50 + \$75.06 + \$254. + \$.09 = ?

Write the sums to be added so that dollars are under dollars and cents under cents. This will bring the decimal points in a column.

Thus:

\$593.60	Notice that when an amount is dollars
75.06	only (as \$254), two naughts are used to
254.00	express the cents. When an amount is
+ .09	cents only (as 9 cents), nothing at all is
\$922.75	written under the dollars.

The numbers are added as if there were no decimal points; but when the adding is completed, write a decimal point in the sum exactly under the other decimal points, so that there will be two figures for cents to the right of the point.

(b) *Problem.* Mr. Benton had \$942.08 in the bank. If he spent \$67.89, how much money had he left?

Solution.

\$942.08	Notice that dollars are written under
− 67.89	dollars, and cents under cents, as in addi-
\$874.19	tion. The decimal points are in a column.

The subtraction is made as if no decimal points were there. In the remainder, the decimal point is placed so

that there will be two figures to the right of it for cents.

\$959.06	\$874.00
− 24.00	− 29.36
\$935.06	\$844.64

Notice that if there are no cents in either the minuend or subtrahend, two naughts are written to the right of the decimal point.

II. DRILL.

(a) \$14.02 + \$9.40 + \$8 = ?
\$116 + \$27.03 + \$19.44 + \$.08 = ?
\$200 + \$5.49 + \$.07 = ?
\$29 + \$18.18 + \$63.70 = ?

(b) \$94.46 − \$59.23 = ? \$58.06 − \$49.19 = ?
\$100 − \$67.34 = ? \$47.12 − \$25 = ?

Continue the drill by dictating further examples until the class can write readily and correctly.

UNCLASSIFIED PROBLEMS

1. In furnishing a room, Mrs. Wilton spent \$67.48 for carpet, \$29.13 for a bureau, \$14 for a dressing table, and \$18.08 for pictures. What was the total cost?

2. For 5 days Henry caught every morning 19 large fish and 7 small ones. How many fish did he catch in the five days?

3. A man had 30 silver dollars which he wished to divide equally among 7 boys. How many did each boy receive, and how many were left over, if none were changed for smaller money?

4. At 4 cents each, how many oranges can Lawrence buy for 30 cents, and how many cents will he have left?

5. A dealer sold a lot for $493.17, another for $674.05, a third for $900 and a fourth for $297.12. What did he receive for them all?

6. In each of 5 bins a farmer had 42 bushels of new corn and 17 bushels of old. How many bushels of corn were in the bins?

7. A man paid 120 dollars for a horse and 47 dollars for a cow. How much cheaper was the cow than the horse?

8. 216 dollars were spent for plate glass windows in a house, at 8 dollars apiece. How many windows were in the house?

9. Find the perimeter (distance around) of a rectangle whose sides are 9 yards and 7 yards?

10. A boy drew a line 17 inches long, and wished to divide it into parts 3 inches long. How many 3-inch parts would there be, and how long would the remaining part be?

11. A horse was bought for $300 and sold for $284.16. What was the loss?

12. A class of 42 pupils was arranged in 5 rows. How many pupils in each row, and how many were not in the rows?

13. A man was absent from home 56 months. During 10 months of this time he was in California, and during 9 months he was in Seattle. The rest of the time he was in Florida. How long was he in Florida?

LESSON FIFTY-TWO

TO MULTIPLY A SUM OF MONEY BY ANY NUMBER

I. EXPLANATION. *Problem.* 64 horses were sold at an average price of $175.07. How much was received for them?

Solution.

$175.07
64 ×
---
700 28
10504 2

$11204.48

Observe that no attention is given to the decimal point until the product is obtained. Then the point is so placed that there will be two places to the right of it for cents.

II. DRILL.

59 × $1435.01 = ?
108 × $1.97 = ?
5274 × $.07 = ?
2500 × 1.63 = ?
49 × $564.10 = ?

REVIEW DRILLS

Place the decimal point in the answers of the following:

$274.16	$864.03	$58.72	$1.46	$ .06
83.00	57.98	36.08	2.00	.57
+ .09	+ 1.40	+4.94	.09	800.00
$357 25	$923 41	$99 74	+3.15	+1.37
			$6 70	$802 00

$586.00	$753.05	$64.28	$3.12
− 72.34	− .09	− 7.56	− .97
$513 66	$752 96	$56 72	$2 15

UNCLASSIFIED PROBLEMS

1. Five hunting dogs were sold for $24.98, $14.06, $19.00, $16.21, $15.00, respectively. What was received for them all?

2. Charles had 42 cents with which to buy 8 tops at the same price. What is the largest number of cents he could pay for each top?

3. Mr. Arthur's cows give 48 gallons of milk every day. How many quarts is that, if there are 4 quarts in a gallon?

4. At a horse sale, a black horse brought $184; a bay one $29.18 less. What was received for the bay one?

5. A drover had 580 cattle. He bought 230 more at one time, and 213 more at another time. How many did he then have?

6. A family uses 3 quarts of cream a week. In how many weeks will it use 132 quarts?

7. 238 ounces of butter will make how many rolls, each weighing 7 ounces?

8. What was paid for 109 geographies at $1.25 each?

9. A teamster had 37 horses. He wished to put them into 9 equal teams. How large a number of horses could be put into each team, and how many would be over?

10. 8 diamonds of equal value were sold for 2056 dollars. How much was that for each?

11. During the first month of this term there were 20 days of school; during the second month there were 18 days of school; and during the third month, 19 days. Warren was absent 9 days during these three months. How many days did he attend school?

12. A rectangular flower bed, 19 feet by 12 feet, was surrounded by a border of primroses. How many feet long was this border?

13. What was the cost of 294 drawing pencils at $.06 apiece?

14. Our milk bill is 6 dollars a month. 27 dollars will pay the bill for how many whole months?

15. A package of ink powder will make 5 pints of ink. How many packages will be needed to make 120 pints of ink?

16. In buying oats for horse feed, a liveryman allowed 46 bushels for each horse. If he kept 38 horses, how many bushels of oats did he buy?

17. In a toy store were 179 china dolls, 234 wax dolls, and 365 rag dolls. Before Christmas 519 of these dolls were sold. How many dolls were in the store after Christmas?

18. 27 bolts of cloth each contained 42 yards. How many yards were there in all?

19. Every day the ice-man leaves 25 pounds of ice at Mr. Smith's, 32 pounds at Mr. Brown's, and 37 pounds at Mr. Harney's. In 5 days how many pounds does he leave at the three houses?

20. What did 74 bars of soap cost at $.08 each?

LESSON FIFTY-THREE

(a) TO DIVIDE A SUM OF MONEY INTO EQUAL PARTS

I. EXPLANATION. *Problem.* 5 books were bought for $11.25. What was the average price?

Solution.

$\frac{1}{5}$ of $11.25 = $2.25. Note that no attention is given to the decimal point until the quotient is obtained. Then two places are pointed off for cents.

(b) TO MEASURE ONE SUM OF MONEY BY ANOTHER, I.E., TO FIND HOW MANY TIMES ONE SUM CAN BE TAKEN FROM ANOTHER

Problem. At 8 cents apiece, how many copy books can be bought for $4.72?

Solution.

$.08)$4.72

59 = number of copy books.

The dividend and divisor must both have two places for cents. The quotient will not be money, and will therefore not have a decimal point.

Until long division is learned, the pupil can not solve problems with a larger divisor than $.12.

II. DRILL. Find:

$\frac{1}{3}$ of $63.27

$\frac{1}{4}$ of $37.00*

$.09)$13.05

$.11)$1.21

(*Notice that two naughts are added for cents.)

$\frac{1}{5}$ of \$128.05 \$.08)\$1.76

$\frac{1}{6}$ of \$103.68 \$.05)\$134.20

UNCLASSIFIED PROBLEMS

1. Fanny had 87 red beads, 46 blue beads, and 63 white beads. She used 94 beads in making a pattern. How many beads had she left?

2. I paid \$11.48 for 4 books. What was the average price?

3. How many straps 9 inches long can be made from a piece of leather 35 inches long?

4. The principal of a school spent 126 minutes in visiting 9 rooms. How many minutes did he average to a room?

5. Mr. Newton sold 64 building lots at an average price (including the making of papers) of \$187.15. What did he receive for them all?

6. The sidewalk in front of 7 equal lots is 301 feet long. How wide is each lot?

7. How many times can 7 pints be taken from 154 pints?

8. A man's gas bills for a year amounted to \$95.64. How much did he pay a month on an average, there being 12 months in a year?

9. A grocer sells a family 4 pounds of butter a week. In how many weeks did the family buy 172 pounds?

10. At 9 cents apiece, how many slates can be bought for \$1.26?

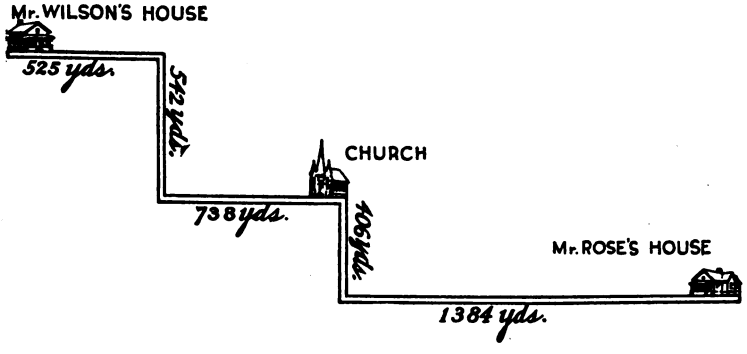
11. 6 cans of milk contain 168 quarts. How many quarts in each can, if they all contain the same number of quarts?

12. Mr. Sawyer had \$807.15 in the bank. He put in \$57.45 more on Monday, and \$9.86 more on Tuesday. On Thursday he drew out \$112.87. How much money had he then in the bank?

13. If 8 gallons of rain water are used every day, how many days will a barrel last which holds 33 gallons?

14. 5874 tablets were sold for 6 cents apiece. How much was received for them?

15. At 4 cents apiece, how many erasers can be bought for \$27.88?



16. Whose house is nearer the church, Mr. Wilson's or Mr. Rose's? How much nearer?

17. Mr. Rose walked from his house to Mr. Wilson's, and then to the church. How far did he walk?

LESSON FIFTY-FOUR

HOW TO SOLVE A CONCRETE PROBLEM REQUIRING TWO OPERATIONS, ONE OF THEM DIVISION

I. EXPLANATION. *Problem.* A merchant had 181 pounds of flour in one barrel, 198 pounds in another, and 179 pounds in a third. If the flour is so placed that each barrel will contain the same quantity, how much flour will be in each?

"State the first problem."

"A merchant had 181 pounds of flour in one barrel, 198 pounds in another, and 179 pounds in a third. *How many pounds in all three barrels?*"

"Think how you would count."

"Write it."

Pupils write,

$$\begin{array}{r} 181 \\ 198 \\ +179 \\ \hline 558 \end{array}$$

"State the second problem."

"*In three barrels there are 558 pounds of flour.* If the flour is placed so that each barrel will contain the same quantity, *how much flour will be in each?*"

"Think how you would count."

"Write it."

Pupils write $\frac{1}{3}$ of $5^{2}5^{1}8 = 186$.

Some pupil is called upon, and says,

"In each barrel there will be 186 pounds of flour."

REVIEW DRILLS

Place the decimal point in the answers:

$56 \times \$1.03 = \$\ 5768$		$\frac{1}{3}$ of $\$\ 7.92 = \$\ 264$	
$9 \times \$21.06 = \$\ 18954$		$\frac{1}{5}$ of $\$18.25 = \$\ 365$	
$17 \times \$9.08 = \$\ 15436$		$\frac{1}{8}$ of $\$24.48 = \$\ 306$	
$4 \times \$.36 = \$\ 144$		$\frac{1}{12}$ of $\$\ 1.08 = 9$	
$159 \times \$.06 = \$\ 954$		$\frac{1}{10}$ of $\$39.20 = 392$	

UNCLASSIFIED PROBLEMS

1. Four boys were hired to distribute handbills. One gave out 247; another 326; another 424; another 205. How many handbills did they distribute altogether?

2. At 9 cents a quart, how many quarts of milk can be bought for $8.19?

3. 4 books at the same price cost 252 cents. How much did each cost?

4. James saved $8.44 one week; $9.67 another week; $5.67 a third week, and $12.59 a fourth week. If a dentist's bill took $20.75 of his savings, how much had he left?

5. In 6 boxes of the same size, 720 oranges were packed. How many were in each box?

6. How many times can 9 bushels be taken from 74 bushels?

7. $37.41 was divided equally among 3 boys. How much did each boy receive?

8. A milliner sold 24 hats at an average price of $13.12. How much did she receive for them?

9. A poultry man shipped to market 94 live Plymouth Rock chickens and 81 Leghorns. He put them

into lath boxes, 5 chickens to the box. How many boxes did he use?

10. A grocer bought 47 boxes of raisins. If the boxes contained 52 pounds apiece, how many pounds of raisins did he buy?

11. From 9 fields, 1125 bushels of wheat were harvested. What was the average number of bushels for each field?

12. Four residences in the suburbs have water tanks. One tank holds 764 gallons, one 843 gallons, one 726 gallons, and one 928 gallons. If one large tank is built to hold as much as the four, how many gallons would it hold?

13. A grocer put 132 pounds of sugar into 4-pound bags. How many bags did he use?

14. A laborer worked 84 days, earning an average of $2.35 a day. How much did he earn in all?

15. A cake contained 12 ounces of flour, 16 ounces of sugar, and 10 ounces of butter. How many ounces of all these materials would 9 such cakes contain?

16. Mr. Johnson receives $325 a month. Mr. Smith receives $175. How much less a month does Mr. Smith receive than Mr. Johnson?

17. In walking 216 inches a man took 8 equal steps. How many inches long was each step?

18. Winston saved 142 cents. He then spent 39 cents, and put the remainder in the savings bank. If he did this 7 times, how much money had he then in the savings bank?

LESSON FIFTY-FIVE

FORM OF LONG DIVISION

I. EXPLANATION. *Problem.* 6816 quarts of plums will make how many bushels, if there are 32 quarts in each bushel?

Imagined counting gives the expression 32)6816, but pupils are unable to compute the answer.

Teacher says, "Let us write it this way, 32)6816(. We will get our answer one figure at a time, and put it to the right." (Or above, if that form is preferred.)

"How many figures in the *divisor* 32?" (This is the first use of the word divisor.)

"Two."

"How many figures in the *dividend* 6816?" (This is the first use of the word dividend.)

"Four."

Teacher draws a line above 68, and says, "We will use this part of the dividend first. Which has more figures, the partial dividend 68, or the divisor 32?"

"Both have the same."

"Very well, then I will say, 'how many times can 3 (the first figure of the divisor), be taken from 6 (the first figure of the partial dividend)?'"

"Two times."

"Yes; then I will put 2 to the right. It is the first figure of the *quotient.*" (This is the first use of the word.)

The blackboard now appears thus, 32)6816(2

"We will multiply the divisor 32 by the quotient figure 2."

"Two times two (touching both figures) are four." (Writing 4 under the 8.)

"Two times three (touching both figures) are six." (Writing 6 under 6.)

"Now we will subtract. Four from eignt leaves four." (Writing the 4.)

The blackboard work now appears thus,

$$\begin{array}{r} 32\overline{)6816}(2 \\ 64 \\ \hline 4 \end{array}$$

"To get a new partial dividend, I bring down 1." (Touches 1 and writes it to the right of the remainder 4.)

"What is my new partial dividend?"

"Forty-one."

"Which has more figures, the partial dividend 41, or the divisor 32?"

"Both have the same."

"Then I will say, 'how many times can 3 (the first figure of the divisor), be taken from 4 (the first figure of the partial dividend)?'"

"Once, and one over."

"What then will be the second figure of the quotient?"

"One."

Teacher writes 1 in the quotient.

"We will now multiply the divisor by the last figure of the quotient, which is one."

"One times two (touching both figures) is two." (Writing 2 under 1.)

"One times three (touching both figures) is three." (Writing 3 under 4.)

"We will now subtract. Two from eleven, nine." (Writing the 9.) "Four from four, nothing."

The blackboard work now appears thus,

$$
\begin{array}{r}
32\overline{)6816}(21 \\
\underline{64} \\
41 \\
\underline{32} \\
9
\end{array}
$$

"To get a new partial dividend, we bring down 6." (Touching 6 and writing it to the right of the remainder 9.)

"What is the new partial dividend?"

"Ninety-six."

"Which has more figures, the partial dividend 96 or the divisor 32?"

"Both have the same."

"Then we will say, 'how many times can 3 (the first figure of the divisor) be taken from 9 (the first figure of the partial dividend)?'"

"Three times."

"Then what will be the next figure of the quotient?"

"Three."

Teacher writes 3 in the quotient, and completes the work by multiplying the divisor by the last quotient figure 3, making the work appear thus,

```
32)6816(213
   64
   ---
    41
    32
    --
     96
     96
     --
```

Teacher reads the problem, "6816 quarts of plums will make how many bushels, if there are 32 quarts in each bushel?"

"How many bushels do we find there are?"

"213 bushels."

"Read the divisor."

"32."

"Read the whole dividend."

"6816."

"Read the quotient."

"213."

"Read the first partial dividend."

"68."

"The second partial dividend."

"41."

"The third partial dividend."

"96."

"How many partial dividends are there?"

"Three."

"How many figures in the quotient?"

"Three."

II. DRILL.

Begin every exercise in arithmetic for the next four days by repeating the above explanation, using one of the following:

(1) Find $\frac{1}{41}$ of 8692.

(2) How many times can 35 be taken from 74,235?

(3) Find $\frac{1}{32}$ of 73,984.

(4) How many times can 23 be taken from 96,922?

UNCLASSIFIED PROBLEMS

1. Have the decimal point placed in the results in the following:

\$506.09	\$957.00	\$643.08
17.48	− 6.18	75×
500.00	\$950 82	3215 40
+ 1.06		45015 6
\$1024 63		\$48231 00

$\frac{1}{5}$ of \$19.26 = 2 1 4

\$.08)\$17.04

2 1 3

2. In one year a farmer feeds 127 pounds of salt to one herd of cattle, 269 pounds to another herd, and 184 pounds to a third herd. How many pounds of salt does he feed to the three herds?

3. A lady had 16 jars of preserved plums, 24 plums in each jar. After the family had eaten 212 of the plums, how many were left?

4. A rectangular park near a large city was 5 miles long and 3 miles wide. A road ran around the outside of it. How long was the road?

5. A storekeeper bought 325 pounds of lard and sold 175 pounds. How many pounds had he left?

6. A merchant had 315 pounds of coffee in 9 sacks. If there was the same quantity in each sack, how many pounds of coffee to the sack?

7. Morgan earned 32 dollars in a week and spent 23 dollars. How much did he save in 29 weeks?

8. How many months in 157 years, if there are 12 months in a year?

9. Last summer our vacation lasted 84 days. We spent 10 of these days going to the mountains, 24 days camping in the mountains, and 10 days returning. The remainder of the time we spent at home. How many days were we at home?

10. At 9 cents apiece, how many rubber balls can be bought for 11 dollars and 16 cents?

11. A letter carrier in 8 hours delivered 336 letters. How many letters an hour did he deliver?

12. Mr. Jones's hens laid 227 eggs in March, 268 eggs in April, and 275 eggs in May. How many eggs did they lay in the three months?

13. Every fourth year is leap year. How many leap years in 104 years?

14. At 24 cents each, what will 39 primers cost?

15. How many hours in 27 days, if there are 24 hours in one day.

LESSON FIFTY-SIX

FORM OF EXPLAINING A CONCRETE PROBLEM IN ADDITION

I. EXPLANATION. *Problem.* John had 26 marbles, Samuel had 24, and George 15. How many had they all?

The imagined counting gives this numerical expression,

$$\begin{array}{r} 26 \\ 34 \\ +15 \\ \hline 75 \end{array}$$

A pupil called on to explain, first reads the problem as given above, and then says,

"They all had 26 marbles, plus 34 marbles, plus 15 marbles, or 75 marbles."

Notice that this explanation is really a statement of the mode of counting, the words "plus" showing that the numbers are all counted from the supply and then counted together.

Hereafter all concrete problems in addition should be explained after the above model, which should be thoroughly learned.

II. DRILL.

Explain the following:

(a) A boy had 15 white rabbits and 19 gray ones. How many rabbits had he?

Solution.

$$\begin{array}{r} 19 \text{ gray rabbits} \\ +15 \text{ white rabbits} \\ \hline 34 \text{ rabbits.} \end{array}$$

(b) In one purse were \$17.56; in another \$14.09, and in a third \$.87. How much money was in the three purses?

Solution.

\$17.56 in 1st purse
14.09 in 2nd purse
\+ .87 in 3rd purse
\$32.52 in the three purses.

(c) 84 pecks of potatoes were dug in one field, 96 pecks in another, and 25 pecks in a third field. How many pecks were dug in all the fields?

Solution.

84 pk. dug in 1st field
96 pk. dug in 2nd field
+25 pk. dug in 3rd field
205 pk. in all the fields.

REVIEW DRILLS

(1)

\$217.14
59.64
+19.00

(2)

\$317.00
−29.06

(3)

\$275.59
234 ×

(4)

$\frac{1}{8}$ of \$21.84 = ?

\$.06)\$25.56

UNCLASSIFIED PROBLEMS

1. A row of corn was 94 feet long. Henry hoed 27 feet of the row. How many feet remained to be hoed?

2. A rose hedge contains 9 bushes of pink roses, 3 bushes of white roses, and 4 bushes of red roses. How many rosebushes in 8 such hedges?

3. 34 lots were sold for $7548. What was the average price?

4. How many times can 32 be taken from 3936?

5. On one of the shelves of a dry-goods store were three rolls of cloth. One roll contained 45 yards of cloth, another 37 yards, and a third roll 42 yards. After 67 yards of cloth were sold, how many yards remained in the three rolls?

6. How many men in 17 companies of militia, if there are 74 men in each company?

7. John visited his Aunt Mary 27 weeks, and his Aunt Alice 9 weeks less than his Aunt Mary. How long did he visit his Aunt Alice?

8. How many times can 41 be taken from 8692?

9. 5 equal bolts of cloth contained 215 yards. How many yards were in each bolt?

10. How many times can 24 be taken from 7536?

11. In one ball of string there were 63 yards, in another 74 yards, and in still another 51 yards. This string was used for flying 4 kites, how many yards of string were used on each kite, if the string was the same length on each?

12. Mr. Samson sold 7 sheep at 6 dollars each. With the money he bought 6 calves. What was the average price paid for each calf?

13. Father bought 3 boxes of red wax candles and 5 boxes of blue ones, each box containing 24 candles. If we used 115 candles to decorate our Christmas tree and house, how many had we left?

14. Find the cost of 92 yards of cloth at $1.12 a yard.

LESSON FIFTY-SEVEN

FRACTIONAL VIEW OF PARTITION

I. EXPLANATION. *Problem.* I have 12 marbles, and wish to divide them equally among 3 boys. How many marbles will each boy receive?

"Into how many equal parts are the marbles divided?"

"Three."

Teacher writes $\overline{3}$.

"How many of these parts does the first boy receive?"

"One."

Teacher writes one over three, thus, $\frac{1}{3}$.

"$\frac{1}{3}$ of what?"

"12 marbles."

Teacher continues the expression, $\frac{1}{3}$ of 12 marbles = ?

"4 marbles."

A few exercises of this kind will serve to shift the emphasis from the mode of counting, (i.e., one at a time into 3 places), to the result of the counting, (i.e., the separation of the group counted into 3 equal parts). The first emphasis is partitive, the second, fractional.

2. DRILL.

(a) Divide 24 cherries into 6 equal parts, and find the value of one part.

(b) Divide 56 books into 8 equal parts. What is one of those parts?

(c) Divide 132 dollars into 11 equal parts. Find one of them.

(d) Find one of the 9 equal parts into which 108 marbles may be divided.

(e) If 36 cents is divided into 9 equal parts, how many cents are in one of those parts?

REVIEW DRILLS

(a) $20{,}675 \times \$1.09 = ?$ (b) $\frac{1}{6}$ of $\$17.82 = ?$

(c) How many times can 8 cents be taken from \$25.92?

UNCLASSIFIED PROBLEMS

1. At an election Mr. Johnson received 874 votes, Mr. Smith 720 votes, and Mr. Wilson 654 votes. How many votes did they all receive?

2. Find $\frac{1}{72}$ of 7992.

3. At an election A received 687 votes, B received 398 votes. How many votes more than B did A receive?

4. Some girls collected 105 roses from one garden, 97 roses from another, and 86 from a third. They made them into bunches of 9 roses each. How many bunches of roses did they make?

5. A club-room contained 27 chairs. If they cost 26 dollars apiece, what did they all cost?

6. A field is 906 feet long and 906 feet wide. How many feet long is the fence which encloses it?

Note. A rectangle whose sides are equal is called a *square.*

7. 598 is how many times 26?

8. A feed chest has 4 bins, each of which will hold 24 pecks of feed. If the chest lacks 38 pecks of being full, how much feed is there in it?

9. Mr. Jones sold 8 cows for 184 dollars. How much was that apiece?

10. What was received for 4864 bushels of wheat at 98 cents a bushel?

11. Find how many times 41 can be taken from 86,961?

12. Mr. Wilson bought a lot, and took 4 years to pay for it. The first year he paid 283 dollars, the second year 329 dollars, the third year 461 dollars, and the fourth year 227 dollars. How much did the lot cost?

13. Find $\frac{1}{31}$ of 71,703.

14. A milliner sold 92 hats during the first week of the season, and had 212 hats remaining. How many hats had she when the season opened?

15. $2875 \times 406 = ?$

16. A farmer fed his hogs 6 bushels of corn each day. In how many days will he feed 432 bushels?

17. Find the perimeter of a rectangle 93 inches long and 64 inches wide. (Draw the figure.)

18. Mr. Brown owed for 9 quarts of ice cream at \$.45 a quart. How much did he owe?

19. Mr. Wilson's bill for milk at \$.08 a quart was \$1.52. How many quarts had he bought?

20. Mr. Johnson bought one lot for \$375.15, and another for \$484. He sold the two for \$1273.96. What did he gain?

21. At 12 cents apiece, how many writing pads can be bought for \$11.28?

22. Mrs. Winston owed the butcher for 5 pounds of beef at \$.27. What was the amount of her bill?

LESSON FIFTY-EIGHT

A NUMERATOR GREATER THAN 1, IN PARTITION

I. EXPLANATION. *Problem.* I divided 12 marbles equally among 3 boys. How many of the marbles did 2 of the boys receive?

"Into how many parts are the marbles divided?"

"Three."

Teacher writes $\overline{3}$.

"How many of these parts did two boys receive?"

"Two."

Teacher writes 2 over 3, thus $\frac{2}{3}$.

"$\frac{2}{3}$ of what?"

"Twelve."

Teacher writes $\frac{2}{3}$ of 12 = ?

Teacher draws a line under this, and then writes $\frac{1}{3}$ of 12 = ?

"Four."

Teacher completes the expression $\frac{1}{3}$ of 12 = 4.

"Then $\frac{2}{3}$ of 12 = ?"

"Eight."

Teacher completes the expression $\frac{2}{3}$ of 12 = 8.

So that the work appears thus,

$$\begin{array}{l} \frac{2}{3} \text{ of } 12 = ? \\ \hline \frac{1}{3} \text{ of } 12 = 4 \\ \frac{2}{3} \text{ of } 12 = 8 \end{array}$$

II. DRILL.

(a) How much are 2 of the 9 equal parts into which 18 pencils may be divided?

(b) 32 cents were divided equally among 8 boys. How many cents did 3 of the boys receive?

(c) If 42 trees were planted in 7 equal rows, how many trees will there be in 5 of these rows?

REVIEW DRILLS

(1) $\$60.93 + \$15.08 + \$294 + \$1.36 = ?$

(2) $1089 \times \$274.05 = ?$

(3) $\frac{1}{11}$ of $\$86.92 = ?$

(4) How many times can $\$.42$ be taken from $\$88.62$?

UNCLASSIFIED PROBLEMS

1. Mary's brother earned 175 dollars one month, 192 dollars the next month, 184 dollars the third month, and 147 dollars the fourth month. How many dollars did he earn in all four months?

2. $\$108.00 - \$74.86 = ?$

3. A rain added 156 quarts to the water in our cistern. After the rain there were 483 quarts in the cistern. How many quarts did it contain before the rain?

4. A gardener picked 44 pecks of peas from one patch, 34 pecks from another, and 26 from another. He sent them to market in baskets holding 4 pecks each. How many baskets did he use?

5. If 35 apples were divided equally among 5 boys, how many did 3 boys receive?

6. $3050 \times \$416.09 = ?$

7. An automobile ran 347 miles one week, 279 the second week, and 426 miles the third week. How many miles did it run during the three weeks?

8. How many times can 21 be taken from 86,772?

9. A builder made a number of houses on the same plan, putting 9 doors in each. He used in all 288 doors. How many houses did he build?

10. Carl studied 39 minutes every evening. How many minutes did he study in 47 evenings?

11. Find $\frac{1}{32}$ of 9952.

12. A certain fish wheel caught one week 534 fish, another week 378 fish, another 475 fish, and the fourth week 630 fish. How many fish did it catch in the four weeks?

13. 12 apples were divided equally among 3 boys. How many apples did 2 of the boys receive?

14. A Brazilian coffee-raiser picked 7963 pounds of coffee. 970 pounds were lost in drying, and the remainder placed in 9 sheds. How many pounds were in each shed, if they all contained the same number?

15. A postoffice sold 512 stamps on Monday, 613 on Tuesday, 596 on Wednesday, and 437 on Thursday. How many stamps were sold on the four days?

16. It took 234 yards of carpet for 6 rooms of the same size. How many yards were needed for each room?

17. 24 marbles were divided equally among 6 boys. What part of the marbles did 5 boys receive? How many marbles did they receive?

18. 16 horses were sold at an average price of $138.25. How much was received for them?

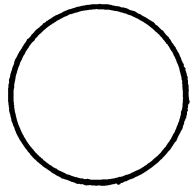
19. How many readers at $.34 each were sold for $7.14?

LESSON FIFTY-NINE

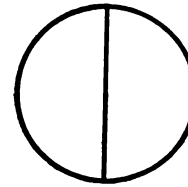
HALVES DEVELOPED OBJECTIVELY

I. EXPLANATION. *Problem.* George had a circular flower garden. He wished to plant roses in one half and geraniums in the other half. Show how he divided it.

Draw the garden in this manner,

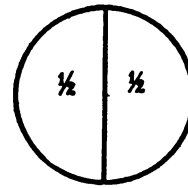


This is how he divided it,



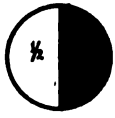
"Into how many equal parts did he divide it?"
"Two."
Show this by writing $\overline{2}$ in each half.
"In how many parts did he plant roses?"
"One."
Show this by writing 1 over $\overline{2}$ in right half circle.
"In how many parts did he plant geraniums?"
"One."
Show this by writing 1 over $\overline{2}$ in left half of circle.

The figure now appears



He had 1 garden; $\frac{1}{2}$ was planted in roses, and $\frac{1}{2}$ was left for geraniums. $1 - \frac{1}{2} = \frac{1}{2}$.

$\frac{1}{2}$ of a garden planted in roses and $\frac{1}{2}$ in geraniums make one garden. $\frac{1}{2} + \frac{1}{2} = 1$.

The figure,  should now be placed on the blackboard in such a position as to form the beginning of the diagram on page 364.

II. DRILL.

(a) A field in the shape of a rectangle was divided into halves by a fence built across it. Draw figure and write numerical expression.

(b) A fishing line was cut into halves. Draw figure and write numerical expression.

(c) A pint of water was divided into two equal parts. Draw the figure and express numerically.

(d) A pound of butter was divided into two equal parts. Draw figure and express numerically. (Use circle.)

REVIEW DRILLS

(1) $673 \times \$.05 = ?$ (2) Find $\frac{1}{14}$ of \$16.94.

UNCLASSIFIED PROBLEMS

1. Find $\frac{3}{7}$ of \$224.

2. A baseball team won 94 games and lost 68. How many games did it play?

3. A farmer sold 80 bushels of potatoes. He then had 224 bushels left. How many bushels had he at first?

4. Jennie spent 65 minutes each day in practicing her music lesson. How many minutes did she practice in 38 days?

5. A peck of oats was divided into 2 equal parts. Draw figure and write numerical expression.

6. A dealer bought 205 pecks of potatoes from one farmer, 306 pecks from another, and 461 pecks from a third. If the potatoes were placed in 4 bins so that each bin had the same number of pecks, how many pecks were in each bin?

7. If it took an average of 7 minutes to mark each examination paper, how many papers did a teacher mark in 294 minutes?

8. Find the value of 288 sheep, worth an average price of $4.15 apiece.

9. Three cans of coal oil contained, one 14, one 17, and one 21 gallons. Part of the oil was sold and the remainder put into a barrel. If the barrel received 27 gallons, how many gallons were sold?

10. A man worked 8 hours a day for 7 days making hogsheads. If it took him 2 hours to make each hogshead, how many did he make in the 7 days?

11. 1304 feet of string were used on 4 kites. If the kite strings were of equal length, how long was each?

12. It requires 171 days to build a certain house, and men have been working on it 147 days. In how many days will the house be finished?

13. 8 horses cost 712 dollars. What did 3 of the horses cost, if all were bought at the same price?

LESSON SIXTY

REDUCTION OF ARTIFICIAL UNITS

(a) *An artificial plurality expressed in one unit, changed so as to be expressed in a smaller unit.*

I. EXPLANATION. *Problem.* A kite string was 139 yards long. How many feet was that?

Imagined counting gives the following:

139 × 3 feet = ?

Compute thus,

$$\begin{array}{r} 139 \times \\ 3 \text{ ft.} \\ \hline 417 \text{ ft.} \end{array}$$

Notice that "1 yard = 3 ft." is an unstated condition of the problem.

II. DRILL.

(a) Change (reduce) 9 lb. to oz.
What is the unstated condition?
(b) Reduce 36 bu. to pk.
What is the unstated condition?
(c) Reduce 29 gal. to qt.
What is the unstated condition?
(d) Reduce 3 yr. to days.
What is the unstated condition?
(e) How many yards in 19 mi.?
What is the unstated condition?

(b) *An artificial plurality expressed in one unit, changed so as to be expressed in a larger unit.*

I. EXPLANATION. *Problem.* A can contains 92 quarts of milk; how many gallons in the can?

"How many quarts are taken out for the first gallon?"
"Four."
"How many for the next gallon?"
"Four."
Write it,

$$4 \text{ qt.})\overline{92 \text{ qt.}}$$
$$23 = \text{no. of gal.}$$

Notice that the fact that 4 quarts equal a gallon is a condition of the problem, but is unstated.

II. DRILL. Give the unstated condition in each of the following:

(a) How many yards in 297 feet?
(b) How many lb. in 192 oz.?
(c) Reduce 296 qt. to gal.
(d) How many weeks in 441 days?

UNCLASSIFIED PROBLEMS

1. A coal dealer bought 723 tons of coal at one time, 563 tons at another time, and 498 tons at still another time. How much coal did he buy in all?

2. Reduce 97 gallons to quarts.

3. Mrs. Johnson and Mrs. Brown went to Europe. Mrs. Johnson stayed 32 months, and Mrs. Brown stayed 61 months. How much longer did Mrs. Brown stay than Mrs. Johnson?

4. 4 pairs of trousers were sold for $31.84. What was the average price?

5. Four oil refineries were owned by the same company. The first refined 3312 barrels of oil in a week, the

second 2489 barrels, the third 4296 barrels, and the fourth 1923 barrels. How many barrels did they all refine?

6. What is the perimeter of a rectangle 94 feet by 67 feet (i.e., 94 feet long and 67 feet wide)?

7. For milk bought at 8 cents a quart, Mr. Watson owes the milkman $13.12. For how many quarts of milk does he owe?

8. To carpet a house required 193 yards of carpet. How many yards of carpet will be needed for 24 such houses?

9. Charles Smith was away from home 17 weeks. How many days was he absent?

10. On a Christmas tree are 6 sacks of candy, with 9 pieces in each sack. If they should be changed so that 6 pieces would be in each sack, how many sacks would be needed?

11. A's gas bill averaged $8.12 a month for 27 months. How much did he pay the gas company in that time?

12. How many times can 32 be taken from 68,192?

13. How many yards long is the sidewalk in front of a lot having 120 feet frontage?

14. 42 chestnuts were divided equally among 7 boys. How many did 3 boys receive?

15. Reduce 184 qt. to pt.

16. How many times can 51 be taken from 5661?

17. A sold some wheat for $5467. After paying bills amounting to $2943.76, how much money had he left?

18. There were 161 people in automobiles. Each automobile carried 7 people. How many automobiles were used?

CHAPTER V

STEPS OF FOURTH YEAR

LESSON SIXTY-ONE

THE FIRST DIFFICULTY IN LONG DIVISION

I. EXPLANATION. *Problem.* How many times can 36 be taken from 7704?

It is easily solved this far,

$$\begin{array}{r} 36\overline{)7704}(21 \\ \underline{72} \\ 50 \\ \underline{36} \\ 144 \end{array}$$

"Which has more figures, the partial dividend or the divisor?"

"The partial dividend."

"Then we say, 'how many times can 3, the first figure of the divisor (touching it) be taken from 14, the first two figures of the partial dividend (touching them)?'"

"Four times."

The remainder of the solution is as follows:

$$
\begin{array}{r}
36\overline{)7704}(214 \\
\underline{72} \\
50 \\
\underline{36} \\
144 \\
\underline{144}
\end{array}
$$

The point especially to be noticed is that when the partial dividend contains one more figure than the divisor, the first *two* figures of the partial dividend are to be used in getting the next figure of the quotient.

UNCLASSIFIED PROBLEMS

1. A dealer sold 82 overcoats for $1179.16. What was the average price?

2. Find the perimeter of a rectangle 42 feet by 23 feet.

3. How many pecks in 184 bushels?

4. How many times can 71 be taken from 80,017?

5. At 74 cents each, how many grammars can be bought for $93.24?

6. 24 peaches were divided equally among 8 boys, how many did 5 of the boys receive?

7. Every day an ice cream maker sends 18 quarts of ice cream to one restaurant, 24 quarts to another, and 32 quarts to another. In 7 days how many quarts of ice cream does he send to the three restaurants?

8. A creamery sold 944 quarts of milk in gallon cans. How many gallon cans were there?

9. 120 inches of cord were needed to hang 5 pictures. How many inches were used for each of the pictures, if the same length of cord was used for each?

10. How many times can 91 be taken from 114,114?

11. A peanut seller roasted 64 pints of peanuts. He sold 30 pints in small packages, and put up the remainder in large sacks, 3 pints to the sack. How many of the large sacks did he have?

12. 3 boxes of soap contained 411 bars. How many bars were in each box, if the boxes all held the same number of bars?

13. 26 boys had each 35 common marbles and 10 alleys. How many marbles did they all have?

14. 63 hats were sold for $134.82. What was the average price?

15. 29 boys earned $1.15 each. How much did they all earn?

16. 1008 cattle were shipped on 8 different days. What was the average number shipped each day?

17. Mary's age is divided into 2 equal parts. Make drawing and write numerical expression. (Time is best represented by a vertical line.)

18. $116.20 was received for hats sold at $4.15 apiece. How many hats were sold?

19. A man took 25 steps each 28 inches long. A boy's steps are 20 inches long. How many steps must the boy take to advance the same distance as the man?

LESSON SIXTY-TWO

FORM OF EXPLAINING A CONCRETE PROBLEM IN SUBTRACTION

I. EXPLANATION. *Problem.* Henry had 43 peaches and ate 9. How many had he left?

Imagined counting gives this numerical expression,

$$\begin{array}{r} 43 \\ -9 \\ \hline 34 \end{array}$$

A pupil being called upon to explain says,

"He had left 43 peaches minus 9 peaches, or 34 peaches."

This is really a description of the mode of counting, the word "minus" showing that the 9 is counted not from the supply, but from the 43.

Hereafter all concrete problems in subtraction should be explained after the above model, which should be thoroughly learned.

DAILY REVIEW DRILLS

1. $34 \times \$186.03 = ?$
2. $\frac{1}{4}$ of $\$1.92 = ?$
3. How many times can \$.08 be taken from \$11.36?

UNCLASSIFIED PROBLEMS

1. It is 184 feet from our front gate to our back fence. From our front gate to the back of our house is 47 feet. How far is it from the back of our house to our back fence?

2. 19 quarts of soda water will fill how many pint bottles?

3. 197 soldiers in camp were divided into three companies. In one company were 72 men, and in another 57. How many men were in the third company?

4. $635 + 826 + 914 + 293 + 175 = ?$

5. 7704 is how many times 36?

6. 48 pounds of candy were put up in ounce packages. How many packages did it make?

7. An oblong field is divided into two equal parts, one part being planted in onions, the other part in radishes. Make drawing and write numerical expression.

8. $6723 + 8468 + 7513 + 2986 + 708 = ?$

9. A storekeeper had 58 gallons of oil in one can, 65 gallons in another, and 33 gallons in a third. If the oil was sold in 6 days, what was the average number of gallons sold each day?

10. A carpet dealer had on hand 175 yards of Brussels carpet and 345 yards of tapestry carpet. It was sold in 8 days. What was the average number of yards sold a day?

11. A tape measure is 16 yards long. How many feet is that?

12. How many times can 54 be taken from 114,912?

13. How many 6-ounce packages can be made from the pepper in 4 cans, one containing 20 ounces, one 25 ounces, one 32 ounces, and one 19 ounces?

14. Mary is 47 years old, and Esther is 29 years younger. Esther has lived in 3 different houses, spend-

ing the same time in each. How long has she lived in each house?

15. On four shelves in a bookcase were 98 books. On the top shelf were 22 books, on the next 27 books, and on the next shelf 28 books. How many books were on the last shelf?

16. A barrel contained 121 quarts of vinegar. Of this 49 quarts were sold, and the remainder placed in demijohns each holding 6 quarts. How many demijohns were needed?

17. Three pieces of ribbon, one 5 yards long, one 7 yards long, and one 4 yards long, were bought at 16 cents a yard. How much was paid for them?

18. A man worked 3 days and 4 hours to build a fence. 8 hours make a day's work. He was paid 45 cents an hour. How much did he get for building the fence?

19. In a school there are 224 boys and 276 girls. If 19 boys and 26 girls should leave, how many pupils would be left in the school?

20. One page of a book contains 29 lines. How many lines will 125 pages contain?

21. A steamship traveled 1956 miles in 6 days. What was the average number of miles traveled each day?

22. A merchant bought 154 yards of cloth. He used 27 yards and sold the remainder at $3.26 a yard. How much did he receive for it?

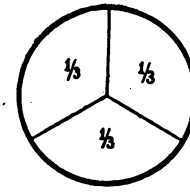
23. 29 horses were sold for $89 each. How much was received for them?

24. $1{,}919{,}887 \times 12 = ?$

LESSON SIXTY-THREE

THIRDS DEVELOPED OBJECTIVELY

I. EXPLANATION. *Problem.* A boy divided his garden into three equal parts, planting geraniums in one part, sweet williams in another part, and wall flowers in the other part. Show how he divided it.

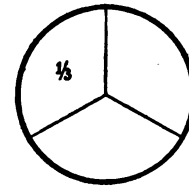


$\frac{1}{3}$ in geraniums and $\frac{1}{3}$ in sweet williams make how many thirds?

$\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

And how much is left for wall flowers?

$1 - \frac{2}{3} = \frac{1}{3}$



The construction of the diagram on the blackboard should be continued by adding the accompanying figure?

II. DRILL.

(a) Divide a rectangular field into 3 equal parts. Make drawing and write numerical expression.

(b) Divide a year into 3 equal parts. (Use vertical line.)

(c) Two knots in a rope divided it into 3 equal parts. Make drawing.

REVIEW DRILLS

1. Each day let one of these be solved:

(a)	(b)	(c)	(d)	(e)
$28.00	$ 163.19	$ 1.09	$ 2.98	$145.08
.98	4274.06	13.46	15.07	29.36
1.34	27.59	27.91	47.18	54.86
+19.75	+ 134.82	+15.64	+36.44	+ 27.39

2. Solve one of the following each day:

(a)	(b)	(c)	(d)	(e)
\$27.64	\$34.29	\$134.17	\$56.94	375 ×
58 ×	1 05 ×	25 00 ×	1 28 ×	\$.06

UNCLASSIFIED PROBLEMS

1. Find $\frac{1}{47}$ of 11,797.

2. A lady bought a yard and a half of ribbon, which was 54 inches. She divided the ribbon among some little girls, giving each 9 inches. How many girls were there?

3. Charlie's mother said he might go out and play for 25 minutes. He has been gone 19 minutes. How soon should he return?

4. A workman saved 14 dollars one month, 12 dollars another, and 16 dollars another. If 27 workmen had saved as much, what would their savings have amounted to?

5. In 7 school months, school was open 133 days. How many days was that for each month?

6. How many times can 48 be taken from 6384?

7. A schoolroom contains 6 rows of 8 desks each, and an extra desk. If the desks had been placed 7 in a row, how many rows would there have been?

8. The perimeter of a rectangle is 60 inches. Its length is 18 inches. What is its width? (Draw figure.)

9. A number of cows were sold for \$4797. The average price was \$39. How many cows were sold?

10. \$638.97 was paid out every week for the wages of 57 people who worked in a factory. What was the average weekly wage of a workman?

11. A board was sawed into 3 equal pieces. Make drawing and write numerical expression.

12. A lady bought 87 inches of Irish lace. She used 48 inches for collars and the rest for trimming handkerchiefs. How much had she for handkerchiefs?

13. 56 ounces of candy were put into 7 packages. How many ounces were in 3 packages?

14. 144 pecks of birdseed are how many bushels?

15. A laborer worked 22 days in March, 24 days in April, 20 days in May, 26 days in June, and 19 days in July. How many days did he work in the five months?

16. A grocer bought 47 boxes of raisins. If the boxes contained 52 pounds apiece, how many pounds of raisins did he buy?

17. A boat traveled 8 miles an hour for 184 miles. How many hours did it take?

18. 24 lots were sold for $400 each. The agent received $20 apiece for selling them. How much did the agent receive for all?

19. 153 cows were put out to pasture for a month. $7 was paid for each cow. How much was paid for all the cows?

20. An apple orchard contains 2187 trees in 9 equal rows. How many trees are in each row?

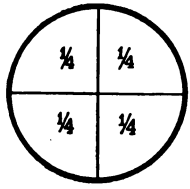
21. A man earns $132 in 11 weeks. How much does he earn in 49 weeks?

22. A farmer sold 7 dozen eggs for $2.45. At the same price, how much would he receive for 294 dozen?

LESSON SIXTY-FOUR

(a) FOURTHS DEVELOPED OBJECTIVELY

I. EXPLANATION. *Problem.* Suppose a boy divides his garden into four parts. Show how he will do it.



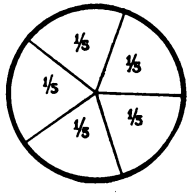
Notice that the garden is first divided into halves; then by drawing a line across it, the halves are divided into fourths.

$$1 - \tfrac{1}{4} = ?$$
$$1 - \tfrac{3}{4} = ?$$
$$1 - \tfrac{2}{4} = ?$$
$$\tfrac{1}{4} + \tfrac{3}{4} = ?$$

The circle showing fourths should be added to the diagram on the blackboard.

(b) FIFTHS DEVELOPED OBJECTIVELY

I. EXPLANATION. *Problem.* Suppose a boy divides his garden into five parts. Show how it will look.



Notice that the garden is divided at once into fifths. There are no larger pieces such as halves or thirds into which it may be divided first.

$$1 - \tfrac{2}{5} = ?$$
$$1 - \tfrac{4}{5} = ?$$
$$\tfrac{2}{5} + \tfrac{3}{5} = ?$$
$$\tfrac{1}{5} + \tfrac{4}{5} = ?$$
$$\tfrac{2}{5} + \tfrac{1}{5} = ?$$

Fifths should be added to the diagram on the blackboard.

REVIEW DRILLS

Dictate one of these each day:

(a)	(b)	(c)
3,040,007	$400.00	$75.97
27,050,964	− 59.08	208 ×
87,009		
+ 198,200		

(d) $\frac{1}{9}$ of $20.88 = ?

(e) $.12)$389.04

UNCLASSIFIED PROBLEMS

1. A man writing a book worked 87 hours one month; 56 hours another month, and 74 hours a third month. How many hours did he work at his book in the three months?

2. During the last two years Mr. James worked 75 weeks. If there are 104 weeks in two years, how long was Mr. James idle?

3. How many times can 69 be taken from 14,904?

4. A jar of candy was divided into 4 equal parts. Make drawing, and write numerical expression.

5. How many hats at 4 dollars apiece must a storekeeper sell in order to receive 132 dollars?

6. Sarah was allowed to remain up 85 minutes in the evening. She studied geography 28 minutes, and arith-

metic 30 minutes. The remainder of the time she read a story book. How long did she read her book?

7. How many times can 56 be taken from 67984?

8. A rope was cut into 5 equal pieces. Make drawing and write numerical expression.

9. A farmer cut 171 tons of hay and put it into stacks, 9 tons to the stack. How many stacks did it make?

10. In 9 boxes of figs there were 144 ounces. How many ounces was that for each box, if all the boxes contained the same number of ounces?

11. A lady made 49 ounces of candy in a kettle. She first took out 10 ounces for taffy, and then divided the remainder equally into three pans to cool. How many ounces of candy were in each pan?

12. 14 yards of cloth were sold for $2 a yard. The money was spent for wood at $4 a cord. How many cords of wood were bought?

13. A city block and the street next it measure 260 feet. How many feet will 23 such blocks and streets measure?

14. How many times can 62 be taken from 1,130,012?

15. A path runs from Mary's house 184 yards across a field, and then 725 yards down the bank of a creek to a footbridge. Mary walked from the house along the path until she was within 134 yards of the footbridge. How many yards did she walk?

16. A farmer dug 565 bushels of potatoes. He sold 100 bushels at once, and stored the remainder in 3 cellars. If the quantity was the same in each cellar, how many bushels did he store in each cellar?

LESSON SIXTY-FIVE

THE SECOND DIFFICULTY IN LONG DIVISION

I. EXPLANATION. *Problem.* Find $\frac{1}{43}$ of 23,134. It is easily solved up to this point,

$$\frac{1}{43} \text{ of } \overline{23134} = 5$$
$$\underline{215}$$
$$163$$

Taking $\frac{1}{4}$ (first figure of divisor) of 16 (first two figures of partial dividend), we get the quotient figure 4; but on multiplying 43 by 4 we obtain 172, which is greater than 163, and can not be subtracted from it. This shows that the last quotient figure is too great, and we must try 3.

The work is then easily completed, thus:

$$\frac{1}{43} \text{ of } \overline{23134} = 538$$
$$\underline{215}$$
$$163$$
$$\underline{129}$$
$$344$$
$$\underline{344}$$

Pupils should now be drilled in multiplying the two figures of the divisor mentally by the last quotient figure, holding the whole product in mind and comparing it with the partial dividend before writing any of it.

When larger divisors are introduced, it will usually be found sufficient to multiply mentally the left two figures

of the divisor by the last quotient figure, and compare the product thus obtained with the corresponding portion of the partial dividend.

Getting the true quotient figure before writing the new product under the partial dividend is the *essential difficulty* in long division. It should be recognized as such and be *drilled upon persistently until mastered.*

REVIEW DRILLS

1. Assign one of the following each day:

(a)	(b)	(c)	(d)	(e)
\$397.06	\$593.19	167 ×	59 ×	6473 ×
2.43 ×	27 ×	\$.19	\$.02	\$4.29

2. Take one of the following each day:

(a) ½ of \$2.94

(b) \$.03)\$1.02

(c) \$.46
.74
.82
+.97

(d) 374,800,007
− 89,310,018

UNCLASSIFIED PROBLEMS

1. How many times can 74 be taken from 18,426?

2. A boy started to visit his cousin. He began the journey by running 225 yards. He then changed his pace to a walk, which he continued for 987 yards further. Finally, he was glad to ride with a farmer the remaining 1284 yards. How far was it from his house to his cousin's?

3. A grocer had 278 pounds of coffee. He sold 193 pounds. How much had he left?

4. Our school year is divided into 10 months of 4 weeks each. If we should divide it into 8 periods, how many weeks would there be in each?

5. 1316 pecks of apples were put into sacks each holding a bushel. How many sacks were used?

6. Find $\frac{1}{29}$ of 11,223.

7. A boy caught during one summer 216 fish. If he caught on an average 9 fish each time he went fishing, how many times did he go fishing that summer?

8. How many days in 25 years, not allowing for leap years?

9. A restaurant keeper bought 97 pounds of butter. 19 pounds spoiled before he used it. How many pounds did he use?

10. A wagon load of coal weighed 1987 pounds. What is the weight of 459 such loads?

11. How many times can 36 be taken from 17,388?

12. From each box of raisins containing 52 pounds, 17 pounds were sold. If there were 12 boxes, how many pounds are left?

13. Anna was absent from home 25 weeks while making a trip to Europe. How many days was she absent?

14. Willie was sick 105 days. How many weeks was that?

15. Find $\frac{1}{57}$ of 16,188.

16. Find the perimeter of a square whose side is 47 inches.

17. A man saves $12 a week. How long will it take him to save $488?

LESSON SIXTY-SIX

HOW TO EXPLAIN A CONCRETE PROBLEM IN PARTITION

I. EXPLANATION. *Problem.* If 17 books of the same kind cost 51 dollars, how much was that apiece?

Imagined counting gives,

$$\tfrac{1}{17} \text{ of } 51 = 3.$$

A pupil being called upon to explain, first reads the problem, and then says,

"Each book costs one-seventeenth of 51 dollars, or 3 dollars."

The pupil must be made to see that this analysis describes the mode of counting, one-seventeenth being a way of saying "one at a time in seventeen places."

(For a development of the fractional meaning of one-seventeenth, see page 214.)

The above model should be thoroughly learned.

II. DRILL. Explain the following:

(a) \$108 was paid for 9 coats. How much was that apiece?

Solution.

$\frac{1}{9}$ of \$108 = \$12.

(b) 5 acres of land cost \$1225. How much was that an acre?

Solution.

$\frac{1}{5}$ of \$1225 = \$245.

(c) 150 books were divided equally in 6 bookcases. How many books were put into each bookcase?

Solution.

$\frac{1}{6}$ of 150 books = 25 books.

REVIEW DRILLS

(a)	(b)	(c)	(d)
$143.65	$783.15	$.57	$147.58
97.28	29.63	.08	29.64
250.97	408.00	.75	8.75
384.62	27.94	1.25	9.12
+ 975.10	.06	+ .36	+ 8.34
	+ 3.21		

Dictate one of the above each day.

UNCLASSIFIED PROBLEMS

1. Fanny had 87 red beads, 46 blue beads, and 63 white beads. She used 94 beads in making a pattern. How many beads had she left?

2. Edgar earned 217 cents, and put 188 of them into his bank. How much did he keep out to spend?

3. A square field is divided into 4 equal parts. Draw and express numerically.

4. How many times can 47 be taken from 35,438?

5. Find the perimeter of a rectangle 48 inches × 34 inches.

6. During last year school was open 196 days. At this rate, how many days would school be open in 27 years?

7. Find $\frac{1}{59}$ of 36,875.

8. Mrs. Rusk is 19 years older than her daughter Mary. If Mrs. Rusk is 36 years old, how old is Mary?

9. A pound of coffee was divided into 3 equal parts. Make drawing and write numerical expression.

10. A feed store sold on an average 95 bushels of oats and 85 bushels of bran a day. How much oats and bran were sold in 26 days?

11. How many times can 38 be taken from 9006?

12. 135 pounds of candles were packed in 3-pound boxes. How many boxes were used?

13. Divide a jar of fruit into 5 equal parts. Draw and express numerically.

14. Columbus discovered America in 1942. Jamestown was founded in 1607. How long after the discovery of America was the founding of Jamestown?

15. Find $\frac{1}{69}$ of 19,182.

16. A confectioner had 48 boxes of figs, each box containing 2 pounds. After he sold 57 pounds, how many pounds had he left?

17. A woman bought a sewing machine for $40. She paid for it in installments of $5 a week. How long did it take her to pay for it?

18. A man bought a farm of 120 acres for $85 an acre. He sold it for $12,000. How much did he gain?

19. A man divided 72 cents equally among 8 boys. What part of the money did 5 of the boys receive? How many cents did they receive?

20. A farmer sold 26 cows at $48 each, and 15 horses at $150 each. How much did he receive for them all?

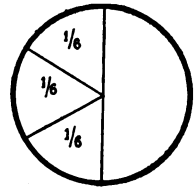
21. John left home on May 10, and returned Oct. 12. How long was he absent?

LESSON SIXTY-SEVEN

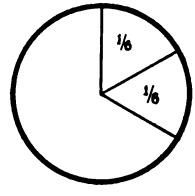
(a) SIXTHS DEVELOPED OBJECTIVELY

I. EXPLANATION. *Problem.* Suppose a boy divides his garden into six parts. Show how it will look.

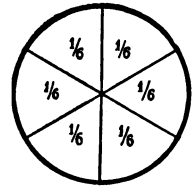
It may be divided first into halves, and then each half may be divided into three equal parts thus:



Or it may be divided into thirds, and then each third be divided into two equal parts thus:



Whichever way is chosen it will look thus when finished:



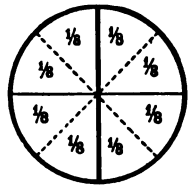
$1 - \frac{2}{6} = ?$
$1 - \frac{3}{6} = ?$
$1 - \frac{5}{6} = ?$
$\frac{4}{6} + \frac{2}{6} = ?$
$\frac{1}{6} + \frac{3}{6} + \frac{2}{6} = ?$

Sixths should be added to the diagram on the blackboard.

(b) EIGHTHS DEVELOPED OBJECTIVELY

I. EXPLANATION. Suppose a boy's garden is to be divided into 8 equal parts.

This can be done by dividing it first into halves, then into fourths, then into eighths, as follows:



The very heavy line is drawn first, the lighter line next, and the dotted lines last.

$$1 - \tfrac{1}{8} = ?$$
$$\tfrac{5}{8} - \tfrac{3}{8} = ?$$
$$\tfrac{6}{8} + \tfrac{2}{8} = ?$$

The circle divided into eighths should now be added to the diagram.

II. DRILL.

(a) Divide a jar of candy into 6 equal parts.
(b) Divide a pint of milk into 8 equal parts.
(c) Divide John's age into 5 equal parts.
(d) Divide a rectangular field into 4 equal parts.
(e) Divide a piece of ribbon into 3 equal parts.
(f) Divide a pound of butter into 8 equal parts.

REVIEW DRILLS

Solve one of these each day:

(a) How many times can 64 be taken from 135,616?
(b) How many times can 56 be taken from 72,128?
(c) How many times can 63 be taken from 264,348?
(d) How many times can 38 be taken from 102,030?

UNCLASSIFIED PROBLEMS

1. A miner found 56 ounces of gold one month, 26 ounces another, 19 ounces a third, and 8 ounces a fourth. How many ounces of gold did he find in the four months?

2. John arose at 18 minutes after 8, and so had 42 minutes to wash, eat breakfast, and go to school. How many minutes did he have for each, if each took the same time?

3. That part of a fence between two posts is called a *panel*. The posts of a certain fence are 6 feet apart, and a wire is stretched across the tops of the posts. If the wire is 192 feet long, how many panels does it cover?

4. There are 52 weeks in a year. If a man works 39 weeks every year and is idle the remainder of the time, how many weeks is he idle in 6 years?

5. Mr. Thompson had 4237 dollars in the bank, and sold a house for 2856 dollars. He then spent 3879 dollars for a farm. How many dollars had he left?

6. The tank of an oil wagon holds 658 gallons. How many gallons will 17 such tanks hold?

7. Mr. Johnson bought a house for 3927 dollars, and sold it for 4274 dollars. How much did he gain?

8. There are 30 days in September, 31 days in October, and 30 days in November. How many weeks in these 3 months?

9. If a boy studies 122 minutes each day, how many minutes does he study in 24 days?

10. A man earned 23 dollars a week. His older son earned 14 dollars a week, and his younger son 8 dollars a

week. The family expenses were 16 dollars a week. How much could they save in a week?

11. If it takes 6 packages of starch, each weighing a pound, to fill a box, how many boxes will 192 pounds of starch fill?

12. 16 boys are flying kites. If each kite string is 227 feet long, how many feet of string are all the boys using?

13. The Thurman school contained 472 boys. Of these 112 did not take part in the Rose Festival parade. The others were placed in ranks with 8 boys in each rank. How many ranks of boys did the Thurman school have in the parade?

14. A storekeeper sold an overcoat and 6 suits of clothes for 184 dollars. He received 46 dollars for the overcoat. There was no difference in the price received for each of the suits of clothes. What was the price of a suit?

15. The width of a rectangle is 16 inches and its perimeter 70 inches. What is its length?

16. How much was received from the sale of 894 shirts at $1.35 each?

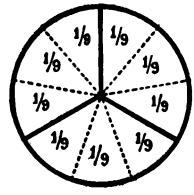
17. Mr. Mason owned three piles of wood. The first contained 15 cords of wood; the second 26 cords, and the third 19 cords. He sold the wood for $6 a cord. How much did he receive for it?

18. A steamer sailed 259 miles the first day, 273 miles the second day, 308 miles the third day, and 319 miles the fourth day. How many miles did she sail in the four days?

LESSON SIXTY-EIGHT

(a) NINTHS DEVELOPED OBJECTIVELY

I. EXPLANATION. This circle is divided first into thirds, then into ninths, the heavy lines being drawn first.



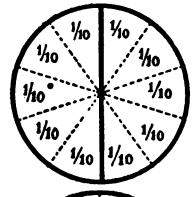
$1 - \frac{7}{9} = ?$
$1 - \frac{5}{9} = ?$
$\frac{4}{9} + \frac{5}{9} = ?$
$\frac{6}{9} + \frac{2}{9} = ?$

Ninths may now be added to the diagram on blackboard.

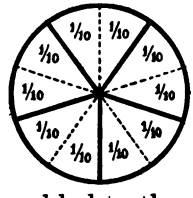
(b) TENTHS DEVELOPED OBJECTIVELY

1. EXPLANATION. A circle may be divided into tenths in two ways.

First. It may be divided into halves, and each half into five equal parts, thus:



Second. It may be divided into fifths, and each fifth into two equal parts, thus:



$1 - \frac{7}{10} = ?$
$1 - \frac{5}{10} = ?$
$\frac{2}{10} + \frac{8}{10} = ?$
$\frac{5}{10} + \frac{3}{10} + \frac{2}{10} = ?$

Tenths may now be added to the diagram on the blackboard.

II. DRILL.

(a) A jar of fruit was divided into 9 equal parts. Make drawing and express numerically.

(b) A fence was divided into 10 equal panels. Make drawing and express numerically.

(c) John's money was divided into 8 equal parts. Make drawing and express numerically.

(d) A period of time was divided into 6 equal parts. Make drawing and express numerically

REVIEW DRILLS

Use one of these each day:

(a) How many times can 49 be taken from 56,105?

(b) How many times can 37 be taken from 71,558?

(c) How many times can 48 be taken from 131,232?

(d) Find $\frac{1}{26}$ of 6604.

UNCLASSIFIED PROBLEMS

1. A long ton of coal weighs 2240 pounds. A short ton weighs 2000 pounds. How much heavier is a long ton than a short ton?

2. The timekeeper's book showed that David Jones worked 164 hours last month; Martin Webster, 156 hours; William Jenkins, 162 hours; Samuel Brown, 147 hours; and Henry Johnson, 150 hours. How many hours did all of the men work?

3. In 6 boxes of soap there are 456 pounds. How many pounds in each box?

4. A piano was sold for 432 dollars. What were 39 pianos sold for at the same price?

5. A circular garden is to be divided into 6 equal parts. Draw the figure in two ways.

6. Mr. Wilson sold a lot for 461 dollars. He put 410 dollars in the bank, and spent the remainder for a set of books at 3 dollars apiece. How many books were in the set?

7. $\frac{4}{9}$ of a circular garden was planted in peas and the remainder in turnips. Draw the figure and write the numerical expression.

8. A farmer bought 6 horses at the same price, and a cow for 60 dollars. He paid for all 552 dollars. What was paid for each horse?

9. $\frac{7}{10}$ of a circular piece of land was cultivated. The rest grew wild. Draw the figure in two ways. Write the numerical expression.

10. A herd of 2085 cattle was sold at an average price of $73.84. How much was received for the entire herd?

11. A florist planted $\frac{3}{8}$ of a circular garden in roses and the remainder in lilacs. Draw the figure and write the numerical expression.

12. A newsboy earned 197 cents one day, 253 cents the next, and 87 cents the third. How many cents did he earn in the three days?

13. Mr. A lived in one house 33 months, in another 51 months, and at a third 42 months. What was the average number of months he lived in a house?

14. Mt. Hood is 11,225 feet high. Mt. St. Helens is 10,000 feet high. How much higher is Mt. Hood than Mt. St. Helens?

15. A can holds 7 pints of syrup. How many such cans will 301 pints fill?

LESSON SIXTY-NINE

HOW TO EXPLAIN A CONCRETE PROBLEM IN MULTIPLICATION

I. EXPLANATION. *Problem.* Mary had 4 pin cushions, with 134 pins in each cushion. How many pins were there in all?

Imagined counting gives this numerical expression:

$$4 \times 134 = ?$$

Which may be written,

$$\begin{array}{r} 134 \\ 4\times \\ \hline 536 \end{array}$$

A pupil called on to explain, first reads the problem, and then says,

"There were 4 times 134 pins, or 536 pins."

This is easily seen to be a description of the mode of counting.

The form should be thoroughly learned and used hereafter to explain problems in multiplication.

II. DRILL. Explain the following:

(a) 19 bins contained an average of 167 bu. of potatoes each. How many bushels did they all contain?

Solution.

$$\begin{array}{l} 167 \text{ bu. in one bin} \\ \underline{19\times = \text{number of bins}} \\ 1503 \\ \underline{167} \\ 3173 \text{ bu. in all the bins} \end{array}$$

(b) The average selling price of 24 houses was $3716.24 each. What was received for them all?

Solution.

$ 3716.24 for one house
24 × = number of houses
14864 96
74324 8
$89189.76 for all the houses

(c) 159 bushels of wheat were sold for 76 cents a bushel. How much was received for the wheat?

Solution.

159 × = number of bushels
$.76 for 1 bu.
9 54
111 3
$120.84 for all the wheat.

REVIEW DRILL

Assign each day one of the following:

(a) How many times can 37 be taken from 22,089?
(b) How many times can 79 be taken from 34,444?
(c) How many times can 68 be taken from 24,344?
(d) How many times can 48 be taken from 24,816?

UNCLASSIFIED PROBLEMS

1. A storekeeper put 257 dollars into the bank on Monday, 754 dollars on Tuesday, 536 dollars on Wednesday, 629 dollars on Thursday, 357 dollars on Friday. How

many dollars did he put into the bank during the five days?

2. 6 girls used a swing 144 minutes. If each of them was in the swing the same length of time, how many minutes did each swing?

3. There are 112 yards of carpet in a roll. How many yards are there in 27 such rolls?

4. A gardener planted $\frac{5}{8}$ of a circular flower bed in geraniums, the remainder in asters. Draw the figure and write the numerical expression.

5. 8 barrels of potatoes contained 3 bushels each. The potatoes were placed in sacks holding 2 bushels each. How many sacks were needed?

6. A real estate dealer sold five lots: one for \$297.83, one for \$483, one for \$516.20, one for \$398, and one for \$973. How many dollars did he receive for all of them?

7. $\frac{3}{5}$ of a piece of string was used for fishing line, the remainder for a top cord. Draw the figure and write the numerical expression.

8. A mile is 5280 feet. A wire fence 6000 feet long is how much longer than a mile?

9. Sold 285 bushels of potatoes at \$1.19 a bushel. How much was received for them?

10. Each of 5 jars contained 22 ounces of candy. If it is put into 6-ounce packages, how many such packages will there be?

11. A ten-acre field produces 357 bushels of oats. How many bushels can be raised on 16 such fields?

12. 37 hats were sold for $220.89. What was the average price?

13. A mill hand worked 298 days one year, 274 the next, 283 the next, and 214 the fourth. How many days did he work in the four years?

14. 5 jars of candy each contained 28 ounces. If 70 ounces of candy were sold from the jars, how many ounces did they still contain?

15. A carpet dealer bought 7 rolls of carpet, each containing 82 yards. He sold 27 yards from each roll. How many yards of carpet had he left?

16. What was paid for 27 tons of coal at $8.25 a ton?

17. A man traveled 153 miles in 9 days. What part of that distance should he travel in 7 days? How many miles should he travel in 7 days?

18. A house was bought for $3987.17 (including cost of papers), and sold for $5000. How much was gained?

19. A clerk spends $560.38 a year and saves $439.62. What is his yearly salary?

20. A farmer received $75.36 for potatoes sold at $1.57 a bushel. How many bushels did he sell?

21. What is the freight on 284 tons of coal at $1.86 a ton?

22. 84 bushels of oats were sold for $32.68. How much was that a bushel?

23. Last month Mary practiced on a piano 104 hours. If she practiced 4 hours each day, how many days did she practice?

LESSON SEVENTY

THE THIRD DIFFICULTY OF LONG DIVISION

I. EXPLANATION. *Problem.* 284,256 is how many times 56?

It is readily solved this far:

$$\begin{array}{r} 56\overline{)284},256(5 \\ \underline{280} \\ 42 \end{array}$$

Notice that the new partial dividend 42, is less than 56, and is, therefore, "not any" times 56.

Then write 0 as the second figure of the quotient.

Now, if we multiply 56 by 0 the product will be 0, and if this be subtracted from 42, the remainder will be 42.

Bringing down the next figure 5, the new partial dividend is 425.

The solution is now easily completed, thus:

$$\begin{array}{r} 56\overline{)284},256(5076 \\ \underline{280} \\ 425 \\ \underline{392} \\ 336 \\ \underline{336} \end{array}$$

Since there are four figures in the quotient, there are four partial dividends, as follows:

284, 42, 425, and 336.

II. DRILL.

In each of the following, name the dividend, the divisor, and the quotient; also each partial dividend, with the quotient figure produced by it.

(a) 83)42,081(507
41 5
581
581

(b) 79)237,316(3004
237
316
316

(c) 27)11,043(409
10 8
243
243

(d) 89)4,450,623(50007
4 45
0 623
623

(e) 93)651,744(7008
651
744
744

(f) 54)32,886(609
32 4
486
486

(g) 54)22,464(416
21 6
86
54
324
324

(h) 39)119,964(3076
117
2 96
2 73
234
234

REVIEW DRILLS

Use one of these each day:

(1) How many times can 84 be taken from 17,556?
(2) How many times can 34 be taken from 70,822?
(3) How many times can 27 be taken from 1,351,728?
(4) How many times can 64 be taken from 409,792?

UNCLASSIFIED PROBLEMS

1. A stock dealer sold five horses: one for 83 dollars, one for 125 dollars, one for 79 dollars, one for 94 dollars, and one for 136 dollars. What did he receive for them all?

2. George bought a bicycle for 15 dollars, and sold it for 9 dollars. How many dollars did he lose?

3. A barrel of flour containing 196 pounds was put into 7 equal sacks. How many pounds were in each sack?

4. In a large mill 846 people are at work. How many will there be at work in 29 such mills, if the same number is at work in each mill?

5. Mr. Williams raised 6000 bushels of wheat this year, and 4864 last year. How much greater was this year's crop than last year's?

6. How many yards long are 201 feet of rope?

7. Find the cost of 87 pounds of butter at 31 cents a pound?

8. Mr. Sanderson worked 17 days one month, 22 another, 25 another, and 20 another. This was equal to how many weeks' work, if there are 6 working days in a week?

9. Each of 5 barrels contained 12 pecks of corn. If the corn had been divided equally among 6 barrels, how many pecks would be in each?

10. A barrel was $\frac{4}{5}$ full of water. Draw figure and write numerical expression.

11. A milk wagon delivered every morning 127 pints of cream. How many pints did it deliver in 38 mornings?

12. Five boys had money banks. In one were 69 cents, in another 241 cents, in a third 48 cents, in a fourth 84 cents, and in a fifth 129 cents. How many cents were in the five banks?

13. 54 calves were bought for \$328.86. What was the average price paid?

14. 325 pounds of sugar contained 86 pounds of sand. How much pure sugar was there?

15. John had his books upon 4 shelves, 18 on each shelf. He rearranged them, using 6 shelves. How many did he put upon each?

16. $\frac{5}{6}$ of a rectangular piece of land was used for a pasture, and the remainder for a vegetable garden. Draw the figure and write the numerical expression.

17. Find $\frac{5}{7}$ of \$29.26.

18. \$158 was received from the sale of neckties at an average price of \$.75. How many neckties were sold?

19. A newsdealer sold 248 papers at 2 cents each. How much did he receive for them?

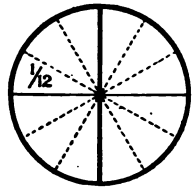
20. At \$7.50 a barrel, how many barrels of flour can be bought for \$510?

LESSON SEVENTY-ONE

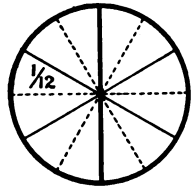
TWELFTHS DEVELOPED OBJECTIVELY

I. EXPLANATION. A circle may be divided into twelfths in three ways:

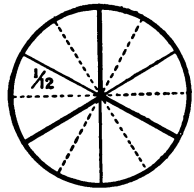
First. Divide into halves, then into fourths, then into twelfths, thus:



Second. Divide into halves, then into sixths, then into twelfths, thus:



Third. Divide into thirds, then into sixths, then into twelfths, thus:



$1 - \frac{1}{12} = ?$
$1 - \frac{5}{12} = ?$
$\frac{9}{12} + \frac{3}{12} = ?$
$\frac{5}{12} + \frac{3}{12} = ?$

UNCLASSIFIED PROBLEMS

1. A party of boys was hired to pick cherries. The first day they picked 108 quarts, the second day 127 quarts, the third day 137 quarts, the fourth day 142 quarts, and the fifth day 148 quarts. How many quarts did they pick in five days?

2. A gardener picked 832 quarts of berries. He sold 756 quarts; the others were bad. How many quarts were bad?

3. 28 horses were sold for $3500. What was the average price?

4. 23 wagons were hauling wheat. Each of them was loaded with 119 pecks of wheat. How many pecks on all the wagons?

5. How many times can 57 be taken from 103,284?

6. How many hours in 365 days?

7. A store sold 564 ounces of candy on Monday, 375 ounces on Tuesday, and 456 ounces on Wednesday. What was the average number of ounces sold each day?

8. Every Monday Mr. Smith walks 5 miles, every Tuesday 6 miles, every Wednesday 8 miles, every Thursday 7 miles, every Friday 9 miles. On Saturday and Sunday he rests. How many miles does he walk in 17 weeks?

9. $\frac{7}{12}$ of a pound of candy was given to a boy, and the remainder of the pound to his sister. Draw the figure in 3 ways. Write the numerical expression.

10. A boy can walk 9 miles a day. How many days will it take him to walk 288 miles?

11. A barrel of flour when full holds 196 pounds. How many pounds have been sold from a barrel which now contains 98 pounds?

12. $\frac{7}{12}$ of a rectangular field was in pasture. The remainder of the field was planted with potatoes. Draw the figure and write the numerical expression.

13. What is the cost of 2087 yards of cloth at \$1.27 a yard?

14. Mr. Smith bought a lot for 400 dollars; spent 1825 dollars for a house, and 215 dollars for improvements. If 24 persons made themselves homes by spending the same sum, what was the cost of the 24 homes?

15. 27 writing desks were sold for \$524.88. What was the average price?

16. In order to drain a swamp, a man had to dig a ditch 125 feet long. There remain 67 feet of the ditch still to be dug. How many feet has the man already dug?

17. 6742 bushels of wheat were sold at \$1.10 per bushel. How much was received for them?

18. 117 acres of land were sold for \$7371. How much an acre was received?

19. At 8 cents a pound, \$75.44 was received for cotton. How many pounds were sold?

20. A sheep raiser sold 2000 sheep. For $\frac{2}{5}$ of them he received \$5.50 apiece. For the others he received \$5.75 apiece. How much did he receive for them all?

21. Last month our gas bill was \$8.70. This month it is $\frac{5}{6}$ as large. How much have we paid for gas for both months?

22. What did our Thanksgiving turkey cost at 26 cents a pound, if it weighed 16 pounds?

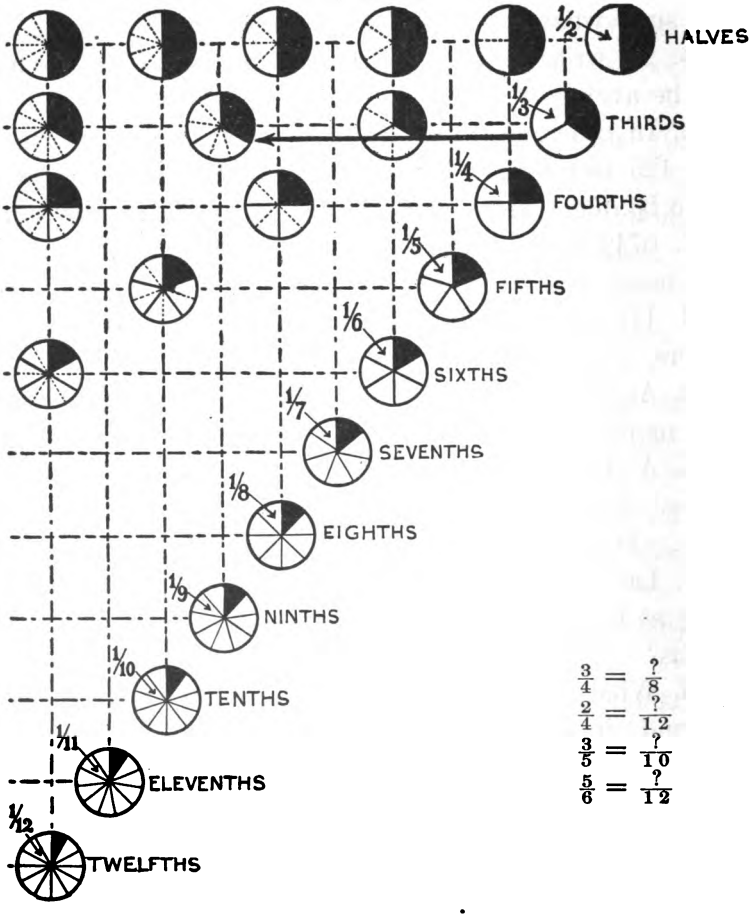
23. A railroad train ran 456 miles in 12 hours. What part of this distance did it run in 7 hours? How many miles did it run in 7 hours?

24. Find the cost of 29 suits at \$17.75 each.

LESSON SEVENTY-TWO

OBJECTIVE REDUCTION OF FRACTIONS TO HIGHER TERMS

I. EXPLANATION. $\frac{2}{3} = \frac{?}{9}$



$\frac{3}{4} = \frac{?}{8}$

$\frac{2}{4} = \frac{?}{12}$

$\frac{3}{5} = \frac{?}{10}$

$\frac{5}{6} = \frac{?}{12}$

Find thirds in the diagram, page 262; pass to the left till ninths are reached on the same line with thirds, and see by counting that $\frac{2}{3} = \frac{6}{9}$.

II. DRILL. Using the diagram, make the following reductions:

$\frac{1}{2} = \frac{?}{4}$	$\frac{1}{4} = \frac{?}{8}$	$\frac{2}{3} = \frac{?}{6}$
$\frac{1}{2} = \frac{?}{6}$	$\frac{3}{4} = \frac{?}{12}$	$\frac{1}{3} = \frac{?}{9}$
$\frac{1}{2} = \frac{?}{8}$	$\frac{1}{5} = \frac{?}{10}$	$\frac{2}{3} = \frac{?}{9}$
$\frac{1}{2} = \frac{?}{10}$	$\frac{3}{5} = \frac{?}{10}$	$\frac{2}{4} = \frac{?}{8}$
$\frac{1}{2} = \frac{?}{12}$	$\frac{1}{6} = \frac{?}{12}$	$\frac{2}{5} = \frac{?}{10}$
$\frac{2}{3} = \frac{?}{6}$	$\frac{5}{6} = \frac{?}{12}$	$\frac{4}{5} = \frac{?}{10}$
$\frac{1}{3} = \frac{?}{12}$	$\frac{3}{8} = \frac{?}{16}$	$\frac{4}{8} = \frac{?}{12}$

UNCLASSIFIED PROBLEMS

1. 6 books of the same kind contain 504 pages. How many pages are in each book?

2. A butcher killed 28 hogs whose weight averaged 194 pounds each. How much did they all weigh?

3. A sheep-raiser had three flocks of sheep. In one flock there were 3295 sheep, in another 1608 sheep, and in the third 2370 sheep. How many sheep had he altogether?

4. How many times can 46 be taken from 93,978?

5. The Declaration of Independence was made 284 years after the discovery of America. The Civil War began 85 years after the Declaration of Independence. How many years was it from the discovery of America to the beginning of the Civil War?

6. Mary had 119 cents saved for Christmas. She spent 42 cents for a doll, 19 cents for some ribbon, and 14 cents for candy. How many cents had she still to spend?

7. How many times can 78 be taken from 111,790?

8. After school Walter plays 125 minutes before getting his dinner. After dinner he studies 47 minutes. How much longer is his play than his study time?

9. One acre of land yielded 36 bushels of wheat, another 42 bushels, another 22 bushels, and a fourth acre yielded 28 bushels. What was the average yield to the acre?

10. A factory produced 327 bolts of cloth. If there were 46 yards in each bolt, how many yards of cloth did the factory produce?

11. Find $\frac{1}{24}$ of 60,144.

12. One cart held 16 pecks of peaches, another 19 pecks, another 23 pecks, and another 14 pecks. How many baskets can be filled with these peaches, if each basket holds 4 pecks?

13. A lady bought a pound of salt, of which 6 ounces were spoiled by rain. The remainder was divided into 5 equal parts to be put into saltcellars. How many ounces of salt were put into each saltcellar?

14. Mr. Walton is now 52 years old, and was 28 years old when his son George was born. How old is George?

15. Change $\frac{3}{4}$ to twelfths.

Change $\frac{4}{5}$ to tenths.

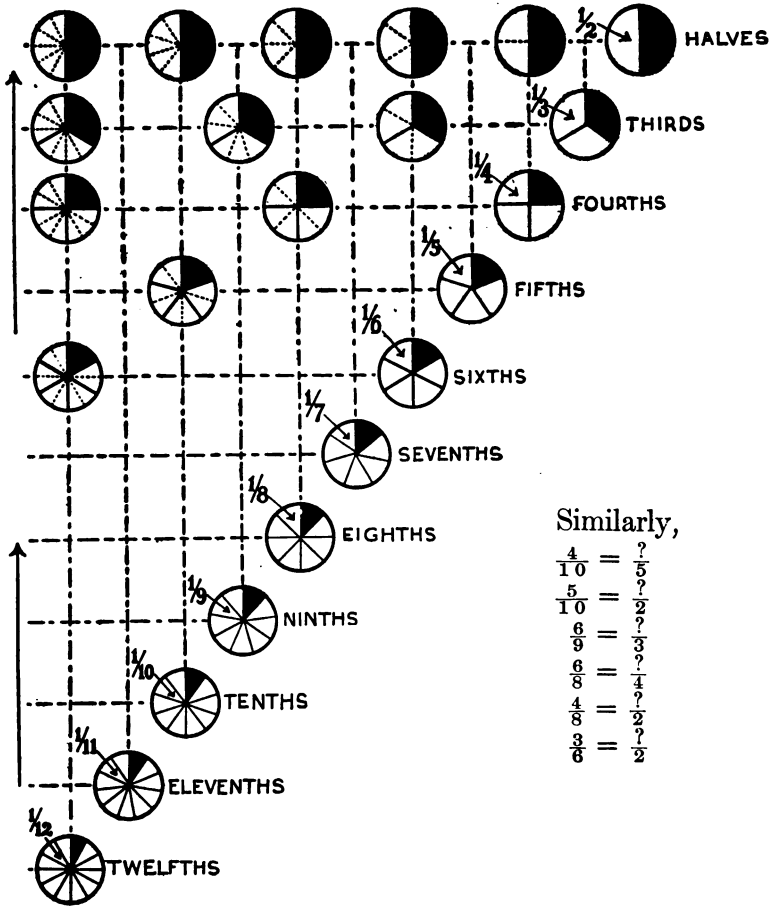
Change $\frac{5}{6}$ to twelfths.

Change $\frac{1}{2}$ to eighths.

LESSON SEVENTY-THREE

OBJECTIVE REDUCTION OF FRACTIONS TO LOWER TERMS

I. EXPLANATION. $\frac{8}{12} = \frac{?}{3}$



Similarly,

$\frac{4}{10} = \frac{?}{5}$

$\frac{5}{10} = \frac{?}{2}$

$\frac{6}{9} = \frac{?}{3}$

$\frac{6}{8} = \frac{?}{4}$

$\frac{4}{8} = \frac{?}{2}$

$\frac{3}{6} = \frac{?}{2}$

Find twelfths in the diagram. Pass upward until the same line with thirds is reached. Then find by counting that,

$$\frac{8}{12} = \frac{2}{3}$$

II. DRILL. Using the diagram, make the following reductions:

$\frac{8}{12} = \frac{?}{3}$	$\frac{6}{9} = \frac{?}{3}$	$\frac{6}{8} = \frac{?}{4}$	$\frac{9}{12} = \frac{?}{4}$
$\frac{5}{10} = \frac{?}{2}$	$\frac{8}{10} = \frac{?}{5}$	$\frac{4}{8} = \frac{?}{2}$	$\frac{8}{12} = \frac{?}{3}$
$\frac{8}{12} = \frac{?}{2}$	$\frac{3}{12} = \frac{?}{4}$	$\frac{4}{12} = \frac{?}{3}$	$\frac{3}{6} = \frac{?}{2}$

UNCLASSIFIED PROBLEMS

1. A farmer has four horses. One weighs 876 pounds, another 1083 pounds, another 1127 pounds, and the fourth 1097 pounds. How many pounds do they all weigh?

2. 9 pounds of candy cost 387 cents. How much was that a pound?

3. A sailor was gone from home 107 weeks. Of this time he spent 15 weeks on land and the remainder of the time at sea. How long was he at sea?

4. A school district bought 27 barrels of ink. Each barrel contained 243 pints. How many pints of ink did the school district buy?

5. Find $\frac{2}{3}$ of 468.

6. Find $\frac{1}{29}$ of 146,218.

7. $\frac{3}{5} = \frac{?}{10}$ $\frac{8}{12} = \frac{?}{3}$.

8. Each of 7 milk cans contains 24 quarts of milk. If the milk be put into pitchers holding 3 quarts apiece, how many pitchers will be needed?

9. A bottling factory bottled 1576 pints of root beer

one week, 2053 pints another week, 1684 pints a third week, and 2279 pints a fourth week. How many pints did it bottle in the four weeks?

10. Find $\frac{1}{37}$ of 125,948.

11. A clothing store sold in one season 1294 suits of clothes, at an average price of \$29.76. For what were all the clothes sold?

12. Find the perimeter of a rectangle 69 inches by 43 inches.

13. 18 demijohns each contained 9 pints of vinegar. If 79 pints were used, how many pints of vinegar remained in the demijohns?

14. (a) Reduce $\frac{9}{12}$ to fourths. (c) Reduce $\frac{1}{3}$ to ninths.
(b) Reduce $\frac{6}{10}$ to fifths. (d) Reduce $\frac{5}{6}$ to twelfths.

15. How many times can 59 be taken from 614,072?

16. It takes 21 days to hatch chickens. If a hen has been sitting 12 days, how soon will the eggs under her hatch?

17. A building 72 feet wide having caught fire, the firemen stretched a rope in front of it. How many yards of rope were needed?

18. At 7 cents a pound, a barrel of sugar cost \$20.79. How many pounds did it contain?

19. Mr. Singer bought a farm for \$5271. He sold it at a loss of \$1482. What did he receive for it?

20. When Mr. Hansen died he left \$67,680 to be divided equally among his 12 children. What part of this money did 9 of the children together receive? How many dollars did they receive?

LESSON SEVENTY-FOUR

HOW TO EXPLAIN A CONCRETE PROBLEM IN MEASUREMENT

I. EXPLANATION. *Problem.* 24 apples were divided among a party of boys, giving each boy 3. How many boys were in the party?

Imagined counting gives this, $3\overline{)24}$ 8

The pupil called upon to explain, first reads the problem, and then says:

"The number of boys equals the number of times that 3 apples can be taken from 24 apples, or 8."

This form of analysis is a better desription of the mode of counting than any of the numerous other forms used.

Let some one form be thoroughly memorized and used hereafter to explain problems in measurement.

II. DRILL. Explain the following:

(a) If soap is sold in boxes holding 6 cakes, how many boxes will hold 54 cakes?

Solution.

$6\overline{)54}$ 9

(b) $110.43 were received for handkerchiefs sold at 27 cents each. How many handkerchiefs were sold?

Solution.

$.27)$110.43(409 = number of handkerchiefs.
108
243
243

(c) At a large department store shirts were marked down to 93 cents. In two days $6517.44 were received from the sale of these shirts. How many were sold?

Solution.

$.93)$6517.44(7008 = number of shirts sold.
651
744
744

(d) 112 young apple trees were planted 8 in a row. How many rows did they make?

8)112
14 = the number of rows.

REVIEW DRILLS

1. Using the diagram, make the following reductions:

(a) $\frac{3}{4}$ to 12ths.
(b) $\frac{8}{10}$ to 5ths.
(c) $\frac{8}{12}$ to 3rds.
(d) $\frac{3}{6}$ to halves.
(e) $\frac{6}{9}$ to 3rds.
(f) $\frac{4}{10}$ to 5ths.
(g) $\frac{1}{3}$ to 6ths.
(h) $\frac{1}{4}$ to 8ths.

2. Use one of the following each day:
(a) How many times can 47 be taken from 8366?
(b) How many times can 54 be taken from 11,729?
(c) How many times can 74 be taken from 9305?
(d) How many times can 87 be taken from 18,211?

UNCLASSIFIED PROBLEMS

1. When he is well Clarence weighs 74 pounds. After a short sickness he weighed 59 pounds. How many pounds did he lose while sick?

2. Grant school has 843 books in its library, Sherman school has 956 books, Johnson school 1067 books, Lee school 972 books, and Washington school 1145 books. How many books are in the libraries of the five schools?

3. 108 boxes of soap with 48 bars to the box were sent to a merchant from the factory. How many bars of soap were sent to him?

4. How many tops at 7 cents apiece can be bought for $1.61?

5. A builder used 168 days in building 3 houses, working the same length of time on each. How many days were spent on each house?

6. Mr. Anson sold 24 calves at 5 dollars apiece. He spent the money for sheep at 6 dollars apiece. How many sheep did he buy?

7. A fish warden put 548 young fish into one stream, 729 into another, and 274 into another. Of these 926 were mountain trout, and the remainder salmon trout. How many salmon trout were there?

8. A storekeeper took in 242 dollars. Of this 90 dollars was for neckties, and the remainder for hats at an average price of 4 dollars apiece. How many hats did he sell?

9. Five lots have, one of them 47 feet front, one 62 feet, one 84 feet, one 27 feet, and one 38 feet. How many feet front have they all?

10. 3 money-banks contained 468 cents. If they all contained the same amount, how much did 2 of the banks contain?

11. From a piece of rope 80 feet long, Mr. Jones cut off 15 feet for the well bucket. He cut the remainder into pieces 5 feet long, to be used to fasten tent flies. How many such pieces did he make?

12. 7 houses were all built on the same plan. They had altogether 168 windows. How many windows were in each house?

13. 1532 quarts of wine were placed in demijohns, each holding a gallon. How many demijohns were used?

14. The ages of 5 men are, 35 years, 43 years, 29 years, 65 years, and 23 years. What is their average age?

15. A pound of candy was put into 6 packages. Make drawing and write numerical expression.

16. A large wheat farm was separated into four divisions. On the first division 12,047 bushels of wheat were raised, on the second 15,922 bushels, on the third 8007 bushels, while on the fourth division 22,789 bushels were raised. How many bushels of wheat were raised on the whole farm?

LESSON SEVENTY-FIVE

LONG DIVISION WITH THREE FIGURES IN THE DIVISOR

I. EXPLANATION. *Problem.* 99,110 are how many times 374?

$$
\begin{array}{r}
374\overline{)99{,}110}(265 \\
\underline{748} \\
2431 \\
\underline{2244} \\
1870 \\
\underline{1870}
\end{array}
$$

The partial dividend and the partial divisor have the same number of figures.

Therefore, to find the first figure of quotient we say, "How many times can 3 be taken from 9?"

3 times, hence try 3 as first quotient figure. 3 times 37 are 111, which is greater than 99; hence 3 is too large. Try 2; 2 times 37 are 74, which is less than 99. Write 2 in the quotient.

Multiply 374 by 2, and subtract the product 748 from the partial dividend, leaving a remainder of 243.

Bring down 1, making a new partial dividend 2431.

To find the next quotient figure say, "How many times can 3 be taken from 24?"

8 times; 8 times 37 are 296, which is greater than 243, Try 7; 7 times 37 are 259, which is greater than 243. Try

6; 6 times 37 are 222, which is less than 243; write 6 in quotient.

Multiply 374 by 6, and subtract the product 2244 from the partial dividend 2431, leaving a remainder 187.

Bring down 0, making a new partial dividend 1870.

To find the next quotient figure say, "How many times can 3 be taken from 18?" Answer 6. But 6 times 37 are 222, which is greater than 187.

Try 5; 5 times 37 are 185, which is less than 187.

Therefore write 5 as the third quotient figure, and multiply 374 by 5. Subtracting the product from the partial dividend, the remainder is 0. Therefore, 99,110 is 265 times 374.

UNCLASSIFIED PROBLEMS

1. Five boys had money-banks. In one were 69 cents, in another 241 cents, in a third 48 cents, in a fourth 84 cents, and in a fifth 129 cents. How many cents were in the five banks?

2. In 6 days a girl answered 294 telephone calls. How many calls was that a day?

3. How many times can 716 be taken from 177,568?

4. There were 46 quarts of molasses in a can. One customer bought 9 quarts, one 16 quarts, and one 12 quarts. How many quarts remained unsold?

5. One barrel contained 115 quarts of vinegar, another 104 quarts, another 123 quarts, and still another 114 quarts. How many jugs each holding 4 quarts may be filled from the 4 barrels?

6. How many inches in 19 feet?

7. A man was 28 years old when his son was born. The son is now 34 years old. How old is the father?

8. How many times can 159 be taken from 1,436,883?

9. A farmer's wife bought 34 yards of cloth to make clothes for the children. When she finished the clothes, 9 yards remained. How much had she used making the children's clothes?

10. How many pounds in 1184 ounces?

11. How many times can 358 be taken from 214,442?

12. From a bolt of cloth containing 36 yards, 19 yards were sold. How many yards remained unsold in 23 bolts of the same size, and from each of which the same number of yards had been sold?

13. A fruit dealer bought 4 boxes of oranges, 72 in a box. He repacked them in 9 smaller boxes, how many did he put into each?

14. 578 neckties were sold at an average price of $1.12. How much was received for them?

15. A farmer sold 7 young cattle at 27 dollars each, and with the money bought sheep at 6 dollars each. How many sheep did he buy, and how many dollars had he left?

16. How many times can 604 be taken from 168,516?

17. $\frac{1}{3} = \frac{?}{12}$; $\frac{3}{4} = \frac{?}{8}$; $\frac{6}{12} = \frac{?}{2}$.

18. A dairyman sold 749 gallons of milk each month. In 24 months, how many gallons did he sell?

19. Four companies of soldiers turned out for drill: one with 74 men, one with 83 men, one with 67 men, and one with 79 men. How many men in the four companies?

CHAPTER VI

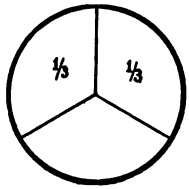
STEPS OF FIFTH YEAR

LESSON SEVENTY-SIX

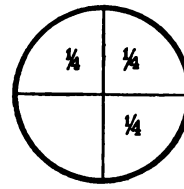
OBJECTIVE ADDITION OF FRACTIONS

I. EXPLANATION. *Problem.* There are $\frac{2}{3}$ of a pound of candy in one jar, and $\frac{3}{4}$ of a pound in another. How much candy in both jars?

Show the candy in the first jar, thus:



Show the candy in the second jar, thus:



From the diagram on page 262 we see that 3rds may be divided into 6ths, 9ths, and 12ths.

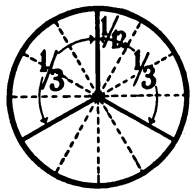
We also see that 4ths may be divided into 8ths and 12ths.

Hence both 3rds and 4ths can be divided into 12ths.

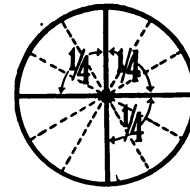
From the diagram we see that,

$\frac{2}{3} = \frac{8}{12}$ and $\frac{3}{4} = \frac{9}{12}$.

Continuing the work, we now have,



$= \frac{2}{3} = \frac{8}{12}$.



$= \frac{3}{4} = \frac{9}{12}$.

$$\frac{8}{12} + \frac{9}{12} = \frac{17}{12} = 1\frac{5}{12}.$$

We take $\frac{4}{12}$ from the $\frac{9}{12}$ and add it to the $\frac{8}{12}$, making 1 pound; then $\frac{5}{12}$ are left where there were $\frac{9}{12}$; so that our answer becomes $1\frac{5}{12}$.

A pupil being called upon to explain says:

"In both jars there were $\frac{2}{3}$ of a pound plus $\frac{3}{4}$ of a pound, or $1\frac{5}{12}$ pounds."

II. Drill. By inspection of the reduction table, answer the following:

(a) Into what can halves and fifths both be divided?
(b) Into what can 3rds and 9ths both be divided?
(c) Into what can 3rds and 6ths both be divided?
(d) Into what can 4ths and 6ths both be divided?
(e) Into what can halves and 4ths both be divided?

REVIEW DRILLS

(a) Find the sum of \$219.75, \$84.25, \$459.07, \$500, \$.94.
(b) From \$700 take \$479.16.
(c) 2806 × \$305.17 = ?
(d) How many times can \$3.75 be taken from \$906.25?

UNCLASSIFIED PROBLEMS

1. In a bookcase containing 6 shelves, there were 19 books on each shelf. If 47 books were removed from the bookcase, how many remained?

2. A lady bought a bolt of flannel containing 37 yards. If she used 29 yards in making undergarments for the children, how many yards of flannel had she left?

3. There were 28 large ships anchored in the harbor. Each had an anchor chain 723 feet long. How many feet of anchor chain were used for all the ships?

4. Find $\frac{1}{56}$ of 1,142,736.

5. A dairyman bought five cows, one for 68 dollars, one for 79 dollars, one for 52 dollars, one for 65 dollars, and one for 73 dollars. How many dollars did they all cost?

6. 356 pecks are how many bushels?

7. A sheep-raiser owned 1536 sheep. He bought 476 sheep at one time, and 257 sheep at another. He then sold 978 sheep. How many had he left?

8. If 36 bureaus were sold for $730.08, what was the average price?

9. John's ruler is $\frac{2}{3}$ of a yard long, and Harry's is $\frac{1}{4}$ of a yard long. What part of a yard are both rulers? Make drawing.

10. A street car conductor is on duty 14 hours each day. How many hours will he work in 274 days?

11. Find $\frac{1}{459}$ of 11,934.

12. Four houses on Tecumseh Street had extension wires attached to their telephones. The first house had 37 inches of wire attached, the second had 42 inches, the third 53 inches, and the fourth 29 inches. How many inches of wire were used to make the extensions?

13. Three men went to a church fair. One had 24 dollars with him, one 19 dollars, and one 26 dollars. They spent among them 37 dollars in chances on a wax doll, and gave the rest to the missionary fund. How many dollars did the missionary fund receive?

14. A chicken raiser had 384 chickens. To prevent the spread of disease, he kept only 24 chickens in each yard. How many yards did he need?

LESSON SEVENTY-SEVEN

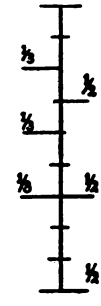
OBJECTIVE SUBTRACTION OF FRACTIONS

I. EXPLANATION. *Problem.* John had $1\frac{1}{2}$ hours for work and play. He worked $\frac{2}{3}$ of an hour. How long did he play?

Draw an hour. Divide it on the right side into halves; add a half. $1\frac{1}{2} = \frac{3}{2}$

On the left side divide the hour into 3rds. Note $\frac{2}{3}$ used for work and the remainder of the $\frac{3}{2}$ hours were used for play. Hence we have,

$$\frac{3}{2} - \frac{2}{3} = \frac{9}{6} - \frac{4}{6} = \frac{5}{6} \text{ for play.}$$



Notice from diagram that $\frac{3}{2} = \frac{9}{6}$ and $\frac{2}{3} = \frac{4}{6}$.

A pupil being called on to explain, first reads the problem and then says,

"He played $1\frac{1}{2}$ hours minus $\frac{2}{3}$ of an hour, or $\frac{5}{6}$ of an hour."

UNCLASSIFIED PROBLEMS

1. Three ice wagons started out in the morning, one with 2143 pounds of ice, another with 2684 pounds, and a third with 1965 pounds. When they came back they had altogether 864 pounds. How much had they delivered?

2. A farmer sold 319 cattle at 42 dollars apiece. What did he receive for them?

3. $\frac{2}{3}$ of a circular garden was planted in flowers, and the rest was in grass. If $\frac{5}{9}$ of the garden was planted in roses, how much of it was planted in other flowers? Make drawing.

4. How many times can 564 be taken from 116,184?

5. Our house tank holds 823 gallons of water. The tank on the house next to us holds only 675 gallons. How much more water does our tank hold than our neighbor's?

6. A druggist sold a certain medicine at 87 cents a bottle. How much did he receive for 109 bottles?

7. $\frac{2}{5}$ of a pound of candy was in one jar, and $\frac{3}{10}$ of a pound in another. How much candy was there in both jars? Make drawing.

8. 2328 drawing erasers came packed in 97 boxes. How many of them were in each box, if all the boxes contained the same number of erasers?

9. 459 young men applied to be soldiers. 27 were rejected because their health was not perfect. The others were divided into 3 equal companies. How many were in each company?

10. How many times can 847 be taken from 171,941?

11. A farmer owned $\frac{1}{5}$ of an oblong piece of land, and bought $\frac{1}{10}$ more of the piece. What part of it did he then own?

12. Mr. Wilson owned $1\frac{1}{2}$ acres of ground, and sold $\frac{5}{8}$ of an acre. What part of an acre had he left?

13. A wholesale grocery house bought 529 pounds of cheese from one factory, 279 pounds from another, 435 pounds from a third, and 179 pounds from a fourth factory. How many pounds did it buy from all?

14. In a potato cellar were five bins. One held 29 bushels of potatoes, one 32 bushels, one 17 bushels, one 27 bushels, and one 19 bushels. How many bushels of potatoes were in the cellar?

15. How many times can 456 be taken from 145,464?

16. A farmer sold his farm for 4160 dollars. He used 1565 dollars to pay a debt, and with the remainder bought 3 city lots at the same price. What did he pay apiece for them?

17. A party of workmen built a piece of road 216 feet long in 6 days. How many feet a day did they average?

18. From $\frac{5}{6}$ of a yard of cloth $\frac{1}{4}$ of a yard was used. What part of a yard remained?

19. In a store were 9 cans of oil, from each of which 17 gallons of oil had been drawn. Each can held at first 35 gallons. How much oil remained in all the cans?

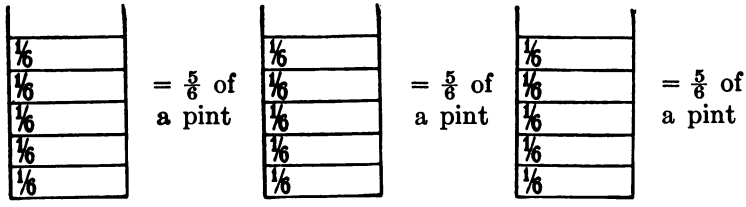
20. Find $\frac{1}{238}$ of 1,030,778.

LESSON SEVENTY-EIGHT

OBJECTIVE MULTIPLICATION OF FRACTIONS

I. EXPLANATION. *Problem.* Each of 3 persons drank $\frac{5}{6}$ of a pint of milk. How many pints did they all drink?

Let us draw what they drank; thus:



By counting it is seen that they all drank $1\frac{5}{6}$ of a pint.

We write it thus, $3 \times \frac{5}{6} = 1\frac{5}{6}$.

If we take $\frac{1}{6}$ of a pint from the left measure and put it into the right measure, then the right measure becomes a pint, while the left becomes $\frac{4}{6}$ of a pint.

We now take another $\frac{1}{6}$ from the left measure and put it into the middle one, making that a pint, and leaving the left measure $\frac{3}{6}$ of a pint.

Writing this, we have $3 \times \frac{5}{6} = 1\frac{5}{6} = 2\frac{3}{6}$.

But from the diagram on page 262, we see that $\frac{3}{6} = \frac{1}{2}$. Hence our numerical expression becomes $3 \times \frac{5}{6} = 1\frac{5}{6} = 2\frac{3}{6} = 2\frac{1}{2}$.

A pupil being called upon to explain says,

"They all drank 3 times $\frac{5}{6}$ of a pint, or $2\frac{1}{2}$ pints."

II. DRILL.

(a) $4 \times 2 = ?$
4×2 hats $= ?$ hats
4×2 tenths $= ?$ tenths
$4 \times \frac{2}{10} = \frac{?}{10}$

(b) $5 \times 3 = ?$
5×3 apples $= ?$ apples
5×3 sevenths $= ?$ sevenths
$5 \times \frac{3}{7} = \frac{?}{7}$

(c) 4×7 eighths $= ?$ eighths
$4 \times \frac{7}{8} = \frac{?}{8}$

(d) 9×8 twelfths $= ?$ twelfths
$9 \times \frac{8}{12} = \frac{?}{12}$

(e) $4 \times \frac{5}{6} = ?$

(f) $9 \times \frac{7}{9} = ?$

REVIEW DRILLS

(a) How many times can 754 be taken from 89,726?
(b) How many times can 156 be taken from 12,168?
(c) How many times can 847 be taken from 171,941?
(d) How many times can 384 be taken from 22,656?

UNCLASSIFIED PROBLEMS

1. A lady 53 years old was 24 years old when her daughter was born. How old is the daughter?

2. There were five barrels of rain water in a laundry shed. The first barrel contained 118 quarts of water, the second 120 quarts, the third 97 quarts, the fourth 112 quarts, and the fifth 93 quarts. How many quarts of rain water were in the shed?

3. It took 8 boxes of the same size to hold 368 pounds of crackers. How many pounds was that to a box?

4. A school board sold 8 lots at 425 dollars apiece. How many school desks could they buy with the money at 4 dollars a desk?

5. In a large field were 384 anthills. If there were on an average 2715 ants in each hill, how many ants were in the field?

6. Mr. Barnes bought 87 cows for 3654 dollars. How much did that average apiece?

7. Henry lived in the country $1\frac{2}{3}$ years. While there he attended school $\frac{3}{4}$ of a year, and worked the rest of the time. What part of a year did he work?

8. In one cart there are 1868 pounds of coal, in another 2146 pounds, and in a third 1944 pounds. How many times may a grate holding 9 pounds be filled from the 3 cart loads?

9. 85 pints of ice cream were bought for a banquet. Only 68 pints were used. How many pints were wasted?

10. It takes 14 inches of lace to make a collar. How much lace will be needed to make 175 collars?

11. At an evening party, there were in one room 5 people whose ages averaged 35 years. In another room were 7 people whose total ages equaled that of the 5 people in the first room. What was the average age of the 7 people?

12. A cabinet maker started to make a piece of furniture for his own use that would take him 40 hours, working 8 hours a day. After working 1 day he was called to another job, and had to finish his own piece of furniture by working 2 hours every evening. How many evenings were required to finish it?

13. A company of militia consists of 1 captain, 2 lieutenants, 4 sergeants, 6 corporals, and 65 privates. How many men in 23 companies?

14. What is the average monthly wages of 4 boys, one of whom receives 27 dollars a month, one 32 dollars, one 47 dollars, and one 38 dollars?

15. A grocer bought 75 ounces of pepper and then had 138 ounces. How many ounces had he before making the purchase?

16. While Mr. Wilson lived in Bridesville, he worked in a factory $\frac{2}{3}$ of a year and was idle $\frac{2}{9}$ of a year. How long did he live at Bridesville?

17. 2496 pounds of soap were put up in boxes containing 48 pounds each. How many boxes were used?

18. A commission merchant sold 148 bushels of potatoes to each of 43 grocery-men. How many bushels did he sell to all of them?

19. An apple orchard produced last year 1200 boxes of apples. $\frac{1}{3}$ of these were of such fine quality that they sold at \$3.15 a box. The others sold for \$1.22 a box. What was received for the entire crop?

20. Henry receives 75 cents a week for spending money. He spends 50 cents of this, and saves the remainder to buy a pair of skates worth \$3.50. In how many weeks will he save enough to pay for the skates?

LESSON SEVENTY-NINE

OBJECTIVE DIVISION OF FRACTIONS

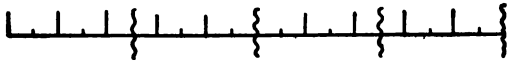
I. EXPLANATION. *Problem.* I have $3\frac{1}{3}$ yards of ribbon, and wish to cut it into pieces $\frac{5}{6}$ of a yard long. How many pieces will there be?

Draw $3\frac{1}{3}$ yards. Notice that it makes $\frac{10}{3}$.

Divide the $\frac{10}{3}$ into 6ths. Notice that it makes $\frac{20}{6}$.

Cut off by a waving line ( ) $\frac{5}{6}$ for each piece, and notic that there are 4 such pieces.

The drawing appears thus:



$$3\tfrac{1}{3} = \tfrac{10}{3} = \tfrac{20}{6}.$$
$$\tfrac{20}{6} \div \tfrac{5}{6} = 4.$$

A pupil being called on to explain, reads the problem and then says,

"The number of pieces will equal the number of times $\frac{5}{6}$ of a yard can be taken from $3\frac{1}{3}$ yards, or 4."

II. DRILL.

10 ÷ 2 = 5.

10 hats ÷ 2 hats = 5.

10 twelfths ÷ 2 twelfths = 5

$\frac{10}{12} \div \frac{2}{12} = 5$

$\frac{9}{8} \div \frac{3}{8} = ?$

$\frac{18}{10} \div \frac{3}{10} = ?$

$\frac{16}{5} \div \frac{4}{5} = ?$

$\frac{20}{3} \div \frac{4}{3} = ?$

$\frac{35}{9} \div \frac{7}{9} = ?$

$\frac{40}{7} \div \frac{5}{7} = ?$

$\frac{45}{12} \div \frac{9}{12} = ?$

$\frac{48}{7} \div \frac{6}{7} = ?$

$\frac{36}{5} \div \frac{4}{5} = ?$

$\frac{18}{7} \div \frac{3}{7} = ?$

REVIEW DRILLS

(a) How many times can 675 be taken from 19,575?
(b) How many times can 246 be taken from 23,370?
(c) How many times can 758 be taken from 158,422?
(d) How many times can 854 be taken from 83,692?

UNCLASSIFIED PROBLEMS

1. In each of 3 jars there was $\frac{7}{8}$ of a pound of candy. How much candy was in three jars?

2. There were 549 gallons of oil in one tank, and 53 gallons less in a smaller tank. How many gallons of oil were in the smaller tank?

3. 357 ounces of flour were used in making 21 cakes of the same size. How many ounces of flour were used in each cake?

4. 22,869 copies of a newspaper were printed every day for 5 days. How many copies were printed during this time?

5. A lady visiting California, used 3 days in traveling there and 4 days in returning. While there she visited 5 cities, remaining 18 days in one, 12 days in another, 15 days in a third, 9 days in a fourth, and 22 days in the fifth. How many days was she away from home?

6. $4\frac{1}{6}$ yards of string were cut into pieces, each of which was $\frac{5}{6}$ of a yard long. How many pieces were made?

7. 7 tanks contain 875 gallons of water each. If 8 gallons of water are used each hour, how many hours will the water last?

8. A flock of wild geese numbering 434 stopped, while on their way south, to rest at a small lake. A party of hunters killed 49 of them. How many were left to continue south?

9. 343 days are how many weeks?

10. $5\frac{1}{4}$ pints of milk were poured into cups, each of which held $\frac{3}{4}$ of a pint. How many cups were filled?

11. A storekeeper had 68 bars of soap on one shelf, and 82 on another. He moved them into drawers, putting 50 bars into each drawer, how many drawers did he use?

12. Harold was allowed 45 minutes every afternoon for play. In 5 days he lost 73 minutes from play because of extra work. How many minutes did he play?

13. Horace spent $\frac{1}{5}$ of a day in work and $\frac{7}{10}$ of the day in play. What part of the day did he use in both work and play?

14. From $1\frac{2}{3}$ yards of ribbon, I cut $\frac{7}{9}$ of a yard to make a bow. How much ribbon remained?

15. A lady bought a bolt of flannel containing 32 yards. After cutting off 11 yards, the remainder was used to make undergarments for 7 children. How many yards apiece did that average?

16. A rich man built 37 houses to cost 2834 dollars apiece. What did he pay altogether?

17. At 19 cents a peck, what will 327 bushels of potatoes cost?

18. A man pays $6 a week board. How much does he pay for board in 52 weeks?

19. 4 baskets of the same size contain altogether 392 apples. What part of that number do 3 baskets contain? How many apples do 3 baskets contain?

20. A bushel of wheat weighs 60 pounds. How many ounces do 249 bushels weigh?

21. A slate 7 inches by 9 inches, is bound with a strip of red flannel. How many inches long is the strip?

22. Show by a circle that $\frac{1}{3}$ of $\frac{1}{4} = \frac{1}{12}$

23. William caught 27 fish, and George caught $\frac{2}{3}$ as many. How many fish did both boys catch?

24. Mr. Fish sold a house for $3500. He spent $\frac{3}{7}$ of the money for a small farm, and put the remainder in the bank. What part of the money did he put in the bank? How many dollars was that?

LESSON EIGHTY

TO REDUCE A MIXED NUMBER TO AN IMPROPER FRACTION (BY COMPUTATION)

I. EXPLANATION. *Problem.* Forty-eight and one-half hours are how many half-hours?

This may be written $48\frac{1}{2}$ hr. $= \frac{?}{2}$ hr.

Solution.

1 hr. $= \frac{2}{2}$ hr.

48 hr. $= 48 \times \frac{2}{2}$ hr. $= \frac{96}{2}$ hr.

$\frac{96}{2}$ hr. $+ \frac{1}{2}$ hr. $= \frac{97}{2}$ hr.

Therefore, $48\frac{1}{2}$ hr. $= \frac{97}{2}$ hr.

The mechanical operation may be written as follows:

$$\begin{array}{l} 48\frac{1}{2} \text{ hr.} \\ \quad 2 \\ \hline 96 \\ \quad 1 \\ \hline 97 \\ \hline 2 \end{array} \text{ hr.}$$

But the oral analysis remains as above.

$48\frac{1}{2}$ hr. is called *a mixed number*, because it *is the sum of a whole number* (48) *and a fraction* ($\frac{1}{2}$).

$\frac{97}{2}$ hr. is called an *improper fraction*, because instead of being *part* of an hour, its value *is greater than one* hour.

The above operation is, therefore, called reducing (changing) a mixed number to an improper fraction.

II. DRILL.

1. Reduce $8\frac{2}{3}$ to an improper fraction.

2. Change $17\frac{5}{8}$ to an improper fraction.

3. $16\frac{3}{4} = \frac{?}{4}$.

4. How many 9ths are in $4\frac{7}{9}$?

5. Find the value of $5\frac{6}{14}$, of $9\frac{7}{8}$, of $13\frac{3}{5}$.

Teacher will dictate mixed numbers to the class and have them reduced to improper fractions until the computation is known to the class.

Pupils should classify the problem when stated in any of the five ways given above.

UNCLASSIFIED PROBLEMS

1. $17\frac{3}{4}$ hours are how many fourths of an hour?

2. (a) $18\frac{4}{5} = \frac{?}{5}$; (b) Reduce $28\frac{5}{6}$ to an improper fraction; (c) Change $75\frac{3}{7}$ to an improper fraction.

3. At 325 dollars each, how many building lots can be bought for 19,756 dollars?

4. Every day Henry spends 45 minutes doing chores, and 95 minutes in his workshop. How many minutes does he work in 42 days?

5. Mr. Johnson bought 63 feet of rope. He used 25 feet for a well rope and 16 feet for a hay wagon. How many feet had he left?

6. A farmer obtained 35 gallons of milk from his cows. 2 gallons were kept for his own use, and the remainder placed in 3 cans. How many gallons were in each can?

7. There was $\frac{3}{4}$ of a quart of milk in one bowl and $\frac{5}{6}$ of a quart in another. How much milk in the two bowls?

8. How many times can 928 be taken from 349,092?

9. How many weeks in 157 years, if there are 52 weeks in a year?

10. A cattle raiser had 374 cattle in one pasture, 183 in another, and 56 in a third. After he sold 416 cattle, how many had he remaining?

11. 14 boys in a class had each 9 sentences to write. Each boy left 3 of his sentences unwritten. How many sentences did all the boys write?

12. 552 pecks of potatoes were divided equally among 23 families. How many pecks did each family receive?

13. A storekeeper pays his bookkeeper 25 dollars a week, one of his clerks 18 dollars a week, the other 15 dollars a week, and the delivery boy 9 dollars a week. How many dollars does he pay out in wages in 27 weeks?

14. A man had $\frac{2}{3}$ of a gallon of cider, and drank $\frac{1}{4}$ of a gallon. What part of a gallon had he left?

15. It is 225 yards from our house to a spring in a field. For 80 yards going from the house there is a gravel path, for 93 yards a dirt path, and for the remaining distance, no path at all. For how many yards is there no path?

16. 4932 pounds of sugar were put into barrels holding 274 pounds each. How many barrels were used?

17. A feed store sold on an average 95 bushels of oats, and 85 bushels of bran a day. How many bushels of oats and bran were sold in 26 days?

18. Each of 3 pint measures was $\frac{7}{12}$ full of water. How much water was there in the 3 pint measures?

19. From one new moon until another is 29 days. How many new moons may be seen in 406 days?

20. There were $2\frac{1}{3}$ yards of cloth in one piece, and $3\frac{5}{6}$ yards in another piece. How many yards were there in both pieces?

21. A dealer in soap received 1567 bars at one time, 2000 bars at another, 1976 bars at another time, and 2143 bars at still another time. He had the soap put up in boxes holding 42 bars each. How many boxes were needed?

22. Major Brown left his daughter 15,964 dollars. This was 3794 dollars less than he left his son. How much did he leave his son?

23. In going to school, Charles goes north on Tenth street 8 blocks, then west on Jackson street 3 blocks, then north on Thirteenth street 2 blocks. How far does he travel, if a block is 310 feet long?

LESSON EIGHTY-ONE

TO REDUCE AN IMPROPER FRACTION TO A WHOLE OR MIXED NUMBER

I. EXPLANATION. *Problem.* How many pounds in 25 half-pounds?

This may be written $\frac{2}{2}^{5}$ pounds = ? pounds.

Solution. The number of pounds is equal to the number of times that $\frac{2}{2}$ pounds can be taken from $\frac{2}{2}^{5}$ pounds or 12 times and $\frac{1}{2}$ pound over.

Therefore, $\frac{2}{2}^{5}$ pounds = $12\frac{1}{2}$ pounds.

The mechanical operation may be briefly written, as follows:

$$\begin{array}{r} 25 \\ \hline 2\overline{)25} \\ \hline 12\frac{1}{2} \text{ pounds.} \end{array}$$

But the oral analysis remains as above.

II. DRILL.

Reduce $\frac{1}{2}^{7}$ to a mixed number.

Change $\frac{3}{7}^{5}$ to a whole number.

In $\frac{6}{9}^{4}$ pounds, how many pounds?

$\frac{5}{7}^{9} = \frac{?}{7}$

Teacher should dictate improper fractions to the class and have them reduced to whole or mixed numbers, until the process is known to the class.

REVIEW DRILL

Reduce $17\frac{3}{4}$ to an improper fraction.

UNCLASSIFIED PROBLEMS

1. A coal dealer bought 552 tons of coal. He sold 164 tons, and placed the remainder in 4 bins. How many tons were in each bin, if the coal was divided equally among them?

2. Reduce $29\frac{5}{8}$ miles to 8ths of a mile.

3. A milk wagon started out with five cans holding 15 gallons, 23 gallons, 19 gallons, 27 gallons, and 24 gallons, respectively. On the way home the unsold milk was poured into one can. If this can contained 37 gallons, how much milk had been sold?

4. (a) In $1\frac{7}{4}$ hours how many hours are there? (b) Reduce $\frac{124}{15}$ to a mixed number.

5. The perimeter of a rectangle is 274 inches and its length 75 inches. What is its width?

6. There are 29 cars in a train. How many such trains will contain 406 cars?

7. For 7 weeks Mary spent $\frac{2}{7}$ of her time practicing her music lessons. How many weeks did she spend in practice?

8. The timekeeper's book at a factory showed that 974 hours had been lost through idleness. If no one had been idle the book would have shown 2520 hours' work. How many hours' work did the book show?

9. A farmer sold 5 horses for 72 dollars each. With the money he bought cows at 40 dollars each. How many cows did he buy?

10. One bag of sugar contained $1\frac{4}{5}$ pounds, and another bag contained $2\frac{3}{10}$ pounds. How many pounds were in both bags?

11. A man bought four houses, one for 4627 dollars, one for 2818 dollars, one for 6217 dollars, and one for 3849 dollars. How much did he pay for them all?

12. William had $1\frac{1}{4}$ pounds of raisins, and gave his brother $\frac{5}{6}$ of a pound. What part of a pound had he left?

13. A gardener raised 15,235 pounds of potatoes. Of these 8067 pounds froze. He put the rest into 56 sacks of the same size. What was the average weight of each sack?

14. 3928 customers paid a gas company an average of \$4.93 a month for gas. What did they all pay?

LESSON EIGHTY-TWO

TO REDUCE A FRACTION TO HIGHER TERMS (BY COMPUTATION)

I. EXPLANATION. In the fraction $\frac{2}{3}$, the 2 and 3 are called the *terms* of the fraction.

The term below the line (3) is called the *denominator*. It names the number of equal parts into which the unit of the fraction is divided.

The term above the line (2) is called the *numerator* of the fraction. It shows how many of the equal parts into which the unit of the fraction is divided are expressed by the fraction.

Problem. Two-thirds are how many twelfths?

This may be written,

$$\frac{2}{3} = \frac{?}{12}$$

$\frac{1}{12}$ is only one-fourth as large as $\frac{1}{3}$.

Therefore, there will be 4 times as many 12ths as 3rds.

$$4 \times 2 = 8$$

Therefore, $\frac{2}{3} = \frac{8}{12}$.

Notice that both terms (2 and 3) of the given fraction have been multiplied by the same number (4), in order to produce the terms (8 and 12) of the new fraction.

Sometimes the process is described as follows:

> *Divide the new denominator by the given denominator, to find what the terms of the given fraction must be multiplied by.*

II. DRILL.

Reduce $\frac{5}{6}$ to 18ths.

$\frac{7}{9} = \frac{?}{36}$

Change $\frac{8}{9}$ to a fraction whose denominator is 27.

Teacher will dictate similar problems until the process of reduction is familiar to the class.

Then have pupils memorize the definitions and rule.

UNCLASSIFIED PROBLEMS

1. (a) Reduce $\frac{3}{4}$ to 12ths. (b) In $\frac{5}{9}$ there are how many 27ths? (c) $\frac{6}{7} = \frac{?}{42}$.

2. (a) Reduce $23\frac{4}{9}$ to an improper fraction. (b) How many 10ths are there in $59\frac{7}{10}$?

3. (a) Reduce $\frac{29}{5}$ to a mixed number. (b) $\frac{18}{7} = ?$

4. $2608 \times \$934.85 = ?$

5. On Monday morning, two customers of a manufacturer paid their bills, one for 156 dollars, the other for 337. The sum of these bills was just enough to pay the weekly wages of his 29 hands, each of whom received the same salary. How much did each hand receive a week?

6. (a) Reduce $\frac{5}{7}$ to 35ths. (b) In $\frac{7}{9}$ there are how many 63rds? (c) $\frac{5}{12} = \frac{?}{48}$

7. (a) Reduce $25\frac{6}{7}$ to an improper fraction. (b) Reduce $\frac{218}{19}$ to a mixed number.

8. If one city has 97,264 people, another 68,528, and a third 106,829, how may people are in the three cities?

9. 239 horses were sold for 20,793 dollars. What was the average price?

10. $1\frac{5}{9}$ pints of milk were put into bottles each holding $\frac{7}{9}$ of a pint. How many bottles were used?

11. (a) $\frac{5}{7} = \frac{?}{49}$ (b) $\frac{6}{8} = \frac{?}{56}$ (c) $\frac{2}{5} = \frac{?}{35}$.

12. A number of boys found that their saving banks contained altogether \$32.64. This was an average of \$1.36 for each bank. How many banks were there?

13. A vinegar factory makes 327 gallons of vinegar in a day. How many gallons can it make in 284 days?

14. A family spent 20 dollars a month for rent, 5 dollars a month for fuel, 25 dollars a month for groceries, 8 dollars a

month for meat, and 9 dollars a month for other expenses. What did it cost them a month to live?

15. A furniture dealer needed 3769 dollars to pay his bills. He had 2017 dollars in the bank. To raise the remainder he had a special sale of writing desks at 24 dollars apiece. How many desks must he sell?

16. (a) $39\frac{3}{4} = \frac{?}{4}$ (b) $\frac{5}{12}\frac{9}{}\frac{7}{} = ?$

17. A business college manager bought 6 typewriters at 88 dollars each. If he had spent the same sum for typewriters at 66 dollars apiece how many could he have bought?

18. A dairyman has 36 cows. If they average 15 quarts a day, how much milk do they all give each day?

19. A factory made in a year 2874 yards of cloth. At the end of the year 386 yards remained on hand. How many yards had been sold?

20. A farmer had 21 acres planted in wheat, and harvested 36 bushels to the acre. How many wagon loads of 42 bushels each did this make?

21. A man bought an automobile, agreeing to pay for it in 15 months, paying 225 dollars a month. Finding this more than he could spare, he had the payments lowered to 135 dollars a month. How many months would it take to pay for the machine at this lower rate?

22. (a) Reduce $\frac{24}{60}$ to its lowest terms. (b) $48\frac{5}{12} = ?$

23. 336 boys belonged to 24 baseball clubs. What was the average number of boys belonging to each club?

24. 84 bushels of beans were divided equally among 7 bins. What part of the beans did 5 bins hold? How many bushels of beans were in the 5 bins?

25. From $\frac{5}{12}$ of 1296 take $\frac{8}{15}$ of 750.

26. Mr. Wilson sold a house for $2963. This was $737 less than the house cost him. What did the house cost him? What did he lose by the sale?

27. Find $\frac{7}{9}$ of 756.

LESSON EIGHTY-THREE

TO REDUCE A FRACTION TO ITS LOWEST TERMS (BY COMPUTATION)

I. EXPLANATION. $\frac{9}{12} = ?$

> In reducing to higher terms, we multiplied both terms of the given fraction by the same number.
>
> In reducing to lower terms we divide both terms of the fraction by the same number.

The question then is, "What number will divide both the terms, 9 and 12?" A mental trial shows 3 to be the common divisor.

Since 3 can be taken from 9 three times, 3 is the new numerator.

Since 3 can be taken from 12 four times, 4 is the new denominator.

That is, $\frac{9}{12} = \frac{3}{4}$.

Notice that each 4th is 3 times as large as a 12th; but there are 3 times as many 12ths as 4ths. Therefore, the fractions are of equal value.

II. DRILL.

$\frac{8}{12} = ?$ (4 is common divisor).
$\frac{15}{25} = ?$ (5 is common divisor).
$\frac{7}{14} = ?$ (7 is common divisor).
$\frac{9}{15} = ?$ (3 is common divisor).
$\frac{27}{36} = ?$ (9 is common divisor).
$\frac{40}{56} = ?$ (what is common divisor?)
$\frac{14}{35} = ?$ $\frac{21}{33} = ?$ $\frac{50}{60} = ?$ $\frac{12}{20} = ?$

Teacher will dictate similar problems until pupils are familiar with the process.

REVIEW DRILL

$\frac{5}{9} = \frac{?}{18}$, $\frac{7}{12} = \frac{?}{60}$, $\frac{5}{9} = \frac{?}{45}$, $\frac{3}{7} = \frac{?}{49}$

$\frac{4}{7} = \frac{?}{56}$, $\frac{3}{8} = \frac{?}{24}$, $\frac{5}{7} = \frac{?}{21}$, $\frac{3}{8} = \frac{?}{72}$, $\frac{5}{16} = \frac{?}{48}$

UNCLASSIFIED PROBLEMS

1. (a) Reduce $\frac{7}{14}$ to $\frac{?}{2}$. (b) Change $\frac{12}{15}$ to 5ths. (c) Reduce $\frac{12}{18}$ to its lowest terms.

2. How many 24ths of a yard are there in $\frac{5}{6}$ of a yard?

3. (a) $83\frac{5}{11} = ?$ (b) Reduce $\frac{138}{8}$ to a mixed number.

4. A contractor wished to buy some horses at a horse sale. He was allowed to choose all he wished at $246 each. If he had $3159 to invest, how many horses could he buy, and how many dollars would he have left?

5. A boy went out to gather chestnuts. He brought home 16 quarts, after selling 15 quarts. How many quarts of chestnuts did he gather?

6. Mr. Wilson sold his house for 5624 dollars. This was 796 dollars more than Mr. Johnson received for his house. For how much was Mr. Johnson's house sold?

7. 377 barrels of oil contain on an average 32 gallons. How many gallons of oil do they contain altogether?

8. 9 barrels of flour contained 1764 pounds of flour. How many pounds was that for each barrel?

9. Find the perimeter of a rectangle 79 inches by 45 inches.

10. 204 overcoats sold at an average price of $14.94. How much was received for them?

11. Find $\frac{1}{675}$ of 19,575.

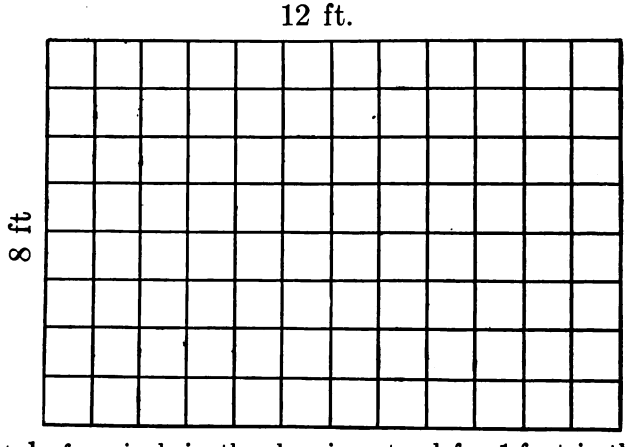
12. 8 bottles of ink were bought for 176 cents. How many cents was that for each bottle?

13. A boy spent 9 hours each day in sleep, 1 hour eating, 5 hours at school, 2 hours working, and the remainder of the time at play. In 5 days, how many hours were there when he was not playing?

LESSON EIGHTY-FOUR

TO FIND THE AREA OF A RECTANGLE

1. EXPLANATION. Draw a rectangle 2 inches wide and 3 inches long, thus:



Let $\frac{1}{4}$ of an inch in the drawing stand for 1 foot in the real figure.

Then this drawing represents a rectangle 8 feet wide and 12 feet long.

On the upper and lower lines of the drawing, mark points $\frac{1}{4}$ inch apart. Connect the points on the upper side with corresponding points on the lower side.

In the same way mark points $\frac{1}{4}$ inch apart on the right and left sides, and connect corresponding points of the two sides.

Notice that the rectangle has been divided into a number of squares, each having sides 1 foot long ($\frac{1}{4}$ in. = 1 ft.).

A square one inch on each side is called a square inch (abbreviated sq. in.).

A square foot (sq.ft.) is a square whose sides are each 1 foot.

Likewise, for every linear unit there is a corresponding square unit.

Notice that the lines just drawn have divided the rectangle into square feet.

Problem. How many square feet in a rectangle 8 feet wide and 12 feet long?

Solution. The answer may, of course, be found by counting the squares, but it may also be computed.

Notice that there are 12 square feet in the row touching the upper side of the figure, the squares in this row being equal in number to the feet in the length of the side.

Notice that there are 8 such rows, the number of rows being equal to the number of feet in the length of the short side of the rectangle.

Therefore, 8×12 sq. ft. $= 96$ sq. ft.

The number of square units in a rectangle is called its *area*.

The rule for computing the area of a rectangle usually given is, "Multiply the length by the width." In interpreting this rule it must be understood that the multiplicand is a number of square units equal to the number of linear units in the length, and the multiplier is an abstract number equal to the number of linear units in the width.

II. DRILL.

Have a rectangle drawn on the blackboard.

(a) Let it be assumed to be 10 feet long and 6 feet wide. Imagine the subdivision into squares and compute the area.

(b) Let the sides of the rectangle be assumed to be 15 feet and 9 feet. Compute its area.

(c) Let the sides of the rectangle be assumed to be 208 yards and 121 yards. Compute its area.

(d) Assume the sides of the rectangle to be 45 feet and 27 feet. Compute its area.

(e) Assume the sides of the rectangle to be 307 feet and 510 feet. Compute its area.

UNCLASSIFIED PROBLEMS

1. Draw a square foot. Divide it into square inches. Compute their number.

2. Draw a square yard. Divide it into square feet. Compute their number.

3. Mr. Kinston receives 7630 dollars a year. Mr. Wright receives 2843 dollars a year. How many dollars a year less than Mr. Kinston does Mr. Wright receive?

4. The money in 5 banks averages 789,364 dollars each. How much money in all the banks?

5. Find the perimeter of a square whose sides are 19 feet. (*A square is a rectangle whose sides are equal.*)

6. $\frac{292}{8} = ?$; $47\frac{5}{9} = ?$; $\frac{3}{12} = \frac{?}{108}$; $\frac{14}{28} = ?$

7. There are 26 working days in a month. David Wilson was idle 7 days each month for 14 months. How many days did he work in that time?

8. Some berry pickers gathered 576 quarts of berries. How many baskets each holding 32 quarts would the berries fill?

9. $\frac{247}{21} = ?$; $48\frac{3}{14} = ?$; $\frac{5}{12} = \frac{?}{96}$; $\frac{36}{81} = ?$

10. A stock-raiser sold on an average 729 cattle each year. How many did he sell in 14 years?

11. At \$2.29 apiece, how many shirts would sell for \$54.96?

12. How many square feet in a garden 50 feet long and 32 feet wide?

13. 111 yards of carpet were used for 3 rooms of the same size. How many yards were used in each room?

14. Find the cost of 15 books at \$2.16 apiece.

15. A field is 283 feet long and 170 feet wide. What is its area? How long is the fence which encloses it?

16. A hogshead holds 63 gallons of oil. How many hogsheads will be needed to hold 1575 gallons?

17. 18,463 + 1975 + 108,359 + 2764 + 836 = ?

18. Find the cost of 29 dictionaries at \$9.75 apiece.

LESSON EIGHTY-FIVE

TO ADD FRACTIONS (BY COMPUTATION)

I. EXPLANATION. *Problem.* There were $\frac{3}{4}$ of a pound of candy in one jar, $\frac{5}{8}$ of a pound in another, and $\frac{1}{6}$ of a pound in another. How much candy was in the three jars?

Solution. This is obviously a problem in addition. But before the fractions can be added, the parts into which the unit of the fraction (a pound) is divided must be the same in each fraction. That is, the fraction must have a *common denominator.*

In reducing fractions to higher terms we found that the new denominator must be divided exactly by the given denominator. Since there are three given denominators (4, 6, and 8), our task is to find the smallest number which can be exactly divided by 4, 6, and 8.

We *mentally* examine the multiplication table, somewhat as follows:

4 will exactly divide 8, 12, 16, 20, 24, 28, 32, etc.,
6 will exactly divide 12, 18, 24, 30, etc.,
8 will exactly divide 16, 24, 32, etc.,

and find that 24 is the smallest number that can be exactly divided by *all* of the given denominators, 4, 6, and 8.

Proceeding as in reduction of fractions to higher terms, we have:

$$\frac{3}{4} = \frac{18}{24}$$
$$\frac{5}{8} = \frac{15}{24}$$
$$\frac{1}{6} = \frac{4}{24}$$
$$\frac{37}{24} = 1\frac{13}{24} \text{ pounds in all the jars.}$$

After reducing to a common denominator, we find by adding the numerators that in the three jars, there are $\frac{37}{24}$ pounds. This is an improper fraction which reduced to a mixed number becomes $1\frac{13}{24}$ pounds.

II. DRILL.

(a) Add $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{5}{6}$.

(b) $\frac{1}{5} + \frac{3}{7} + \frac{9}{35} = ?$

(c) Find the sum of $\frac{1}{6}$, $\frac{5}{9}$, and $\frac{2}{3}$.

Teacher will dictate similar problems until pupils are familiar with the process.

UNCLASSIFIED PROBLEMS

1. Mary had two pieces of blue ribbon. One of them was $\frac{5}{6}$ of a yard long, and the other $\frac{1}{4}$ of a yard long. How long were the two pieces together?

2. (a) Reduce $7\frac{5}{12}$ to an improper fraction. (b) Reduce $\frac{21}{28}$ to its lowest terms.

3. Mrs. Jones uses 125 pounds of ice each week. Mrs. Williams uses 212 pounds. Mrs. Smith uses 178 pounds. How many pounds do they all use in 9 weeks?

4. William Johnson's standings at the end of last term were: Reading 87, Geography 79, Arithmetic 82, Spelling 92, and Writing 67. What was his average standing?

5. A hop raiser picked 18,975 pounds of hops. 465 pounds were unfit for use. The remainder sold for 23 cents a pound. What did he receive?

6. Find $\frac{1}{436}$ of 29,212.

7. At the end of the month, a city treasurer had on hand 108,932 dollars. Out of this he had to pay 47,246 dollars for salaries. How much was left for other expenses?

8. Building lots in a certain addition to the city were sold at 455 dollars apiece. If the addition contained 246 lots, and all were sold, how much was received for them?

9. Three pieces of braid were respectively $\frac{5}{6}$ of a yard long, $\frac{3}{10}$ of a yard long, and $\frac{4}{5}$ of a yard long. How long were they all?

10. A miner found 56 ounces of gold one month, 26 ounces another, 19 ounces a third, and 8 ounces a fourth. How many ounces of gold did he find in the four months?

11. In 4 bins there were 972 bushels of wheat. What was the average number of bushels in a bin?

12. There are 24 hours in a day. If Burton slept 9 hours every day for 27 days, how many hours was he awake in that time?

13. An apple orchard produced 3694 bushels of apples one year, 2787 bushels the next, 4205 bushels the next, and 3463 bushels the fourth. How many bushels of apples did the orchard produce in four years?

14. Mr. Winston lived on a farm on which he owed 2564 dollars. His father gave him 3572 dollars. He paid what he owed on his farm, and with the remainder bought 18 Jersey cows. What did he pay apiece for his cows?

15. (a) How many 14ths in $78\frac{3}{14}$? (b) Reduce $\frac{36}{48}$ to its lowest terms.

16. A stock raiser had 210 cattle. 48 of them are in a large pasture. 27 of the others are in each of his smaller pastures. How many of the smaller pastures are there?

17. A school spent \$3145 for typewriters at \$85 each. How many typewriters were bought?

18. A lot 50 feet by 100 feet was sold for \$2500. How much was that a square foot?

19. If 2 pounds of sugar cost 18 cents, what will 15 pounds cost?

20. A farm of 86 acres was bought for \$45 an acre and sold for \$4000. How much was gained?

21. A storekeeper sold on Monday goods worth \$45.73; on Tuesday goods worth \$62.91; on Wednesday, Thursday, and Friday the daily sales averaged \$47.83. How much money was taken in on the five days?

22. After spending \$19.65, a man had \$4.94. How much had he at first?

23. 18 tons of hay were sold for \$255.60. How much was that a ton?

LESSON EIGHTY-SIX

TO ADD MIXED NUMBERS (BY COMPUTATION)

I. EXPLANATION. *Problem.* In one tank there were $108\frac{3}{4}$ gallons of water; in another tank there were $96\frac{3}{8}$ gallons; in a third tank there were $87\frac{7}{16}$ gallons. How many gallons of water were in the three tanks?

Solution. The operation is obviously addition.

$108\frac{3}{4}$	12
$96\frac{3}{8}$	6
$87\frac{7}{16}$	7
+	
$292\frac{9}{16}$	$\frac{25}{16} = 1\frac{9}{16}$

Commencing at the right, we add the fractions first.

A mental inspection (perhaps trial) shows the common denominator to be 16, which we write once in the position shown in the figure.

$\frac{3}{4} = \frac{12}{16}$. We write the numerator 12 as in the figure.

$\frac{3}{8} = \frac{6}{16}$. We write the numerator 6 as in the figure.

$\frac{7}{16} = \frac{7}{16}$. We write the numerator 7 as in the figure.

Adding the new numerators, we obtain 25 as the numerator of the sum of the fractions.

This we write over the common denominator 16 (already written), as shown in the figure.

$\frac{25}{16}$ being an improper fraction, we reduce it to the mixed number $1\frac{9}{16}$.

$\frac{9}{16}$ is written under the original column of fractions, as shown in the figure.

The whole number 1 is carried to the units' column.

Adding the whole numbers, we find their sum to be 292.

Therefore, in the three tanks there were $292\frac{9}{16}$ gallons of water.

II. DRILL.

Teacher will dictate problems in addition of mixed numbers until the class is familiar with the operation. This may require all or part of several periods.

UNCLASSIFIED PROBLEMS

1. $307\frac{5}{6}$ bushels of wheat grew in one field, $586\frac{3}{4}$ bushels in another field, and $429\frac{2}{3}$ bushels in a third field. How many bushels of wheat grew in the three fields?

2. (a) In $\frac{15}{18}$ of a pound, how many 6ths of a pound are there?
(b) $29\frac{4}{15} = \frac{?}{15}$

3. \$189.37 was received from the sale of trousers at an average price of \$6.53 a pair. How many pairs were sold?

4. People promised to give 57,684 dollars to build the baby home, but 11,367 dollars of this was never paid. How much money was on hand to build the home?

5. Two gold prospectors dug 259 ounces of gold every month until they had dug 4144 ounces altogether. How long did it take them?

6. A rectangle is $18\frac{3}{4}$ inches by $9\frac{5}{6}$ inches. What is its perimeter?

7. A ship sailed 329 miles one day, 275 the next day, 193 miles the third day, and 364 miles the fourth day. How far did it sail in the four days?

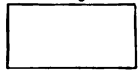
8. 483 suits of clothes were sold for 14,007 dollars. What was the average price of a suit?

9. Mr. Hillis paid 45 dollars a month rent. Mr. Davis paid 29 dollars a month rent. In 42 months, how much more rent will Mr. Hillis pay than Mr. Davis?

10. Mr. Anson lived in one house $2\frac{1}{3}$ years, and in another $1\frac{1}{4}$ years. How long did he live in both houses?

11. In 58 equal steps a man moved forward 1624 inches. How many inches long was each step?

12. 157 men earned on an average 84 dollars a month. What did they all earn in a month?

13.  Find the area of a rectangle 18 feet by 15 feet.

14. A jeweler received 1986 dollars from the sale of gold cuff buttons, at 8 dollars a pair. How many pairs did he sell?

15. On a farm were three wheat fields of 10, 15, and 22 acres, respectively. From the first field 257 bushels of wheat were harvested, from the second field 312 bushels, and from the third field 512 bushels. What was the average number of bushels to the acre?

16. 84,257 × $6.70 = ?

17. A merchant sold 8 barrels of flour for $50.24. What was the price of a barrel?

18. A man received 35 cents an hour for his work. His son received 14 cents an hour. How much more than his son did the father receive for 94 hours' work?

19. A farmer sold one week 28 dozen eggs at 35 cents a dozen. The next week he sold 22 dozen at 41 cents a dozen. How much did he receive for his eggs in both weeks?

20. A doctor's automobile travels on an average 26 miles a day. How far does it travel in 119 days?

21. A man spoke at the rate of 152 words a minute. How long did it take him to make a speech of 6384 words?

22. A storekeeper sold 18 yards of silk at $1.95 a yard. What did he receive for it?

23. A school bought 42 pupils' desks at $4.50, a teacher's desk at $12.80, a clock at $6.25, 3 chairs at $1.25, and a dictionary at $8.25. What was paid for all of these articles?

24. A dealer received $72.32 for potatoes at $1.13 a barrel. How many barrels did he sell?

LESSON EIGHTY-SEVEN

TO DRAW A TRIANGLE AND FIND ITS PERIMETER

I. EXPLANATION. *Problem.* A three-cornered field is 108 feet on one side, 54 feet on another side, and 81 feet on the third side. How many feet will a man walk in going round the field. (Draw figure.)

Solution. Let 1 inch on the paper represent 27 feet on the ground.

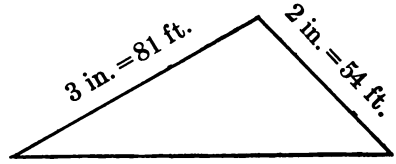
Then the number of inches on the paper needed to represent 108 feet on the ground will equal the number of times 27 feet can be taken from 108 feet, or 4.

That is,

27 ft.)108 ft.(4 = number of inches for long side.
108

27 ft)81 ft.(3 = number of inches for next side.
81

27 ft.)54 ft.(2 = number of inches for short side.
54



If the lower side is drawn first, it will require a compass to locate exactly the upper corner, but it can be placed very near the proper position by measuring with a rule.

The man will walk,

$$\begin{array}{r} 108 \text{ ft.} \\ 81 \text{ ft.} \\ +\ 54 \text{ ft.} \\ \hline 243 \text{ ft.} \end{array}$$

II. DRILL.

Teacher will assign various values to the sides of the triangle already drawn, and have the perimeter computed until the process is familiar.

UNCLASSIFIED PROBLEMS

1. The three sides of a triangle are 17 feet, 27 feet, and 37 feet. What is its perimeter?

2. Mr. Ranson saved \$564 one year, \$614 another, \$497 a third, \$715 in a fourth year, and \$659 in a fifth year. What were his average yearly savings for this period?

3. One barrel of flour contained $178\frac{5}{8}$ pounds, another barrel contained $184\frac{1}{6}$ pounds, another $192\frac{1}{3}$ pounds. How many pounds of flour did all of the barrels contain?

4. One side of a rectangular flower bed is $46\frac{2}{3}$ feet. The other side is 20 feet. How many feet long is a cement curb surrounding the flower bed?

5. (a) Reduce $\frac{77}{132}$ to its lowest terms. (b) Reduce $184\frac{3}{16}$ to an improper fraction.

6. How many times can 476 be taken from 103,643?

7. On Monday the maximum (highest) temperature was 79 degrees, on Tuesday it was 85 degrees, on Wednesday 83 degrees, on Thursday 93 degrees, and on Friday 96 degrees. What was the average maximum temperature for the 5 days?

8. Every day a meat company sends 3968 pounds of meat to one market, 2547 pounds to another market, and 4562 pounds to a third. How much meat does the company send to the three markets in 27 days?

9. Draw a triangle whose sides are 17 feet, 19 feet, and 30 feet. Find its perimeter.

10. A picture frame, measured on the outside, is 17 inches long and 14 inches wide. How long was the piece from which the frame was made? (Draw figure.)

11. Five apple orchards averaged 812 boxes of first grade apples, 409 boxes of second grade apples, and 374 boxes of third grade apples. How many boxes of apples of all kinds did the five orchards produce?

12. $27{,}754{,}375 \div 6875 = ?$

13. $20\frac{7}{9} + 114\frac{2}{3} + 29\frac{4}{15} + 8\frac{2}{5} = ?$

14. Find the area of a rectangle 265 feet $\times$ 74 feet.

15. How many times can 374 be taken from 1,097,316?

16. A wood yard sold 109 loads of wood one day, 187 loads another day, 98 loads another day, 156 loads a fourth day, and 135 loads on the fifth day. Find the average number of loads sold per day.

17. A man took 75 full steps averaging 28 inches each. How many feet did he advance?

18. A man left \$33,468. Of this \$500 was to be given to his church. The remainder was to be divided equally among his 4 children. How much did each child receive?

19. One barrel of flour contains $192\frac{3}{8}$ pounds; another $187\frac{1}{4}$ pounds; another $194\frac{5}{6}$ pounds, and a fourth $191\frac{7}{12}$ pounds. How much flour do the four barrels contain?

20. It required $24\frac{3}{5}$ yards of carpet for one room, $22\frac{7}{10}$ for another, and $19\frac{1}{5}$ for a third. How many yards of carpet were needed for all of the rooms?

21. 8 men earned \$98.32. If each earned the same amount, what part of the money did 5 men earn? How much money did they earn?

22. How much will 48 silver spoons cost at \$12.65 a dozen?

23. One family eats $14\frac{5}{6}$ pounds of meat in a week; another family eats $12\frac{3}{4}$ pounds. How much do both families eat?

LESSON EIGHTY-EIGHT

TO SUBTRACT FRACTIONS (BY COMPUTATION)

I. EXPLANATION. *Problem.* William passed $\frac{5}{6}$ of an hour in work and play together. If he played $\frac{3}{4}$ of an hour, what part of an hour did he work?

Solution. The problem is subtraction. The common denominator is 12.

$$\begin{array}{r} \frac{5}{6} = \frac{10}{12} \\ -\ \frac{3}{4} = \frac{9}{12} \\ \hline \end{array}$$

$\frac{1}{12}$ hr. Therefore, he worked $\frac{1}{12}$ of an hour.

II. DRILL.

Teacher will dictate problems in subtraction of fractions until pupils are familiar with the operation.

UNCLASSIFIED PROBLEMS

1. (a) $\frac{3}{4} - \frac{5}{8} = ?$ (b) $\frac{5}{6} - \frac{1}{3} = ?$ (c) $\frac{7}{9} - \frac{4}{7} = ?$

2. (a) Reduce $\frac{90}{270}$ to its lowest terms. (b) Reduce $23\frac{5}{18}$ to an improper fraction.

3. On one kite there are $156\frac{5}{8}$ feet of string. On another kite there are $87\frac{2}{3}$ feet, on a third kite $129\frac{5}{6}$ feet, and on a fourth kite $80\frac{7}{9}$ feet. How many feet of string are on the four kites?

4. How many times can 796 be taken from 42,984?

5. Find the perimeter of a rectangle $96\frac{1}{2}$ yards by 80 yards.

6. 357 soldiers in a camp were divided into 5 companies. In one company were 72 men, in another 64, in another 59, in a fourth 82. How many soldiers were in the fifth company?

7. Find the perimeter of a triangle whose sides are 25 feet, 20 feet, 16 feet.

8. In a school of 5 rooms, there are 40 pupils in one room, 36 in another, 48 in another, 37 in another, and 52 in the fifth room. What is the average number of pupils to a room?

9. How many times can 526 be taken from 106,778?

10. What is the area of a rectangle 54 inches by 29 inches?

11. (a) Reduce $5\frac{5}{24}$ to an improper fraction. (b) How many 17ths in $54\frac{1}{17}$?

12. In one purse were $\$42\frac{3}{4}$, in another $\$27\frac{1}{2}$, in a third $\$17\frac{3}{20}$, and in a fourth $\$9\frac{4}{10}$. How many dollars were in the four purses?

13. $209\frac{5}{6} + 87\frac{4}{9} + 96\frac{2}{3} = ?$

14. Find the perimeter of a triangle whose sides are 34 inches, 20 inches, 17 inches.

15. How many times can 465 be taken from 13,485?

16. One field contained $15\frac{5}{12}$ acres, another $17\frac{1}{6}$ acres, another $5\frac{3}{4}$ acres. How many acres were in the three fields?

17. A dealer sold 45 cows for \$1890, and 18 cows for \$670.32. What was the average price for which the cows were sold?

18. John earns as much money in 24 days as William earns in 36 days. If William receives \$2 a day, how much a day does John receive?

19. At a place in a city where three streets come close together there is a small park in the shape of a triangle. The three sides are $87\frac{1}{8}$ feet, $96\frac{5}{12}$ feet, and $40\frac{1}{6}$ feet. How long is the iron fence around the park?

20. A factory employs 98 hands and pays them each month \$5684. What was the average monthly wages of the workmen?

21. A real estate dealer sold a lot for \$954. This was \$215 more than it cost. What did the lot cost?

22. 125 hats were sold for \$518.75. What was the average price of a hat?

23. What is the largest number of automobiles at \$3500 each that can be bought for \$36,780?

LESSON EIGHTY-NINE

(a) TO SUBTRACT ONE MIXED NUMBER FROM ANOTHER WITHOUT CARRYING

I. EXPLANATION. *Problem.* A hogshead of molasses contained $59\frac{7}{8}$ gallons. If $48\frac{4}{5}$ gallons were sold, how many gallons remained in the hogshead?

The operation is subtraction.

$$\begin{array}{r|l} 59\frac{7}{8} & 35 \\ -48\frac{4}{5} & 32 \\ \hline 11\frac{3}{40} & \frac{3}{40} \end{array}$$

The operation is so similar to addition of mixed numbers that an inspection of the figure will make it clear without further explanation.

II. DRILL.

Teacher will dictate problems until pupils are familiar with the process.

(b) TO SUBTRACT ONE MIXED NUMBER FROM ANOTHER WHEN CARRYING IS NECESSARY

Suppose a problem requiring

$$\begin{array}{r|l} 108\frac{2}{5} & 14 \\ -\ 76\frac{6}{7} & 30 \\ \hline 31\frac{19}{35} & \frac{19}{35} \end{array}$$

Proceeding as in the last case, we have $\frac{30}{35}$ to subtract from $\frac{14}{35}$.

Since this is impossible, we take one of the 8 units and add it to the $\frac{14}{35}$, making $1\frac{14}{35}$, or $\frac{49}{35}$.

Subtracting $\frac{30}{35}$ from $\frac{49}{35}$, there remain $\frac{19}{35}$.

We have taken 1 from 8, and have 6 more to take, making in all 7 from 8, which leaves 1.

Completing the subtraction, we find the remainder to be $31\frac{19}{35}$.

II. DRILL.

Teacher will dictate abstract problems until pupils are familiar with the process.

UNCLASSIFIED PROBLEMS

1. From $49\frac{5}{7}$ pounds of flour, $37\frac{9}{14}$ pounds were used. How much remained?

2. A merchant bought a number of barrels of flour, each of which contained $194\frac{1}{8}$ pounds. He took $7\frac{4}{5}$ pounds from each barrel. How many pounds of flour did each barrel then contain?

3. What is the area of a rectangle 56 inches by 39 inches?

4. 275 horses were sold for $50,875. What was the average price?

5. A storekeeper had $59\frac{3}{7}$ gallons of molasses in one hogshead, and $47\frac{2}{14}$ gallons in another. He sold $63\frac{1}{3}$ gallons. How many gallons remained?

6. Find the perimeter of a triangle whose sides are respectively 13 feet, 14 feet, and 15 feet.

7. In building a barn $50\frac{3}{4}$ feet long, and $27\frac{5}{6}$ feet wide, a carpenter first drove four stakes, one at each corner, and connected them with a long cord. How long was the cord? (Draw figure.)

8. How many times can 476 be taken from 103,463?

9. $307\frac{5}{6} - 89\frac{8}{9} = ?$

10. 507 straw hats were sold at an average price of $.73. What was received for them?

11. A merchant bought 246 hats, but found 94 of them so badly damaged that he returned them to the seller. The remaining hats were sold at the rate of 8 a day. How many days did it take to dispose of them?

12. A rectangle is $37\frac{1}{5}$ inches long and $27\frac{4}{15}$ inches wide. How much does the length exceed the width?

13. $5276\frac{1}{3} + 893\frac{4}{7} + 1086\frac{5}{7} - 4397\frac{2}{3} = ?$

14. A storekeeper had $59\frac{3}{7}$ gallons of molasses in one hogshead, and $47\frac{2}{14}$ gallons in another. He sold $63\frac{1}{3}$ gallons. How many gallons remained?

15. An ice cream factory furnished to each of 17 stores 42 quarts of vanilla ice cream, 38 quarts of strawberry ice cream, and 16 quarts of chocolate ice cream. How many quarts of ice cream of all kinds did it sell to all the stores?

16. A farmer sold his hay for $1069. He used $329 in settling his store bill. With the remainder he bought 4 horses. What was the average price of the horses?

17. In each of 2 months there were 26 working days. John Merton was idle 4 days each month. How many days did he work? How much did he earn at $3.15 a day?

18. A book has 32 lines on each page. If the book has 278 pages, how many lines does it contain?

19. It cost $4434.75 to seat the assembly hall in a high school with opera chairs costing $3.65 each. How many people can be seated in the assembly hall?

20. A dealer brought $22\frac{3}{8}$ quarts of vanilla ice cream and $17\frac{1}{4}$ quarts of strawberry ice cream. He sold $29\frac{5}{8}$ quarts of both kinds. How many quarts remained unsold?

21. At $6.85 a ton, what will 27 tons of coal cost?

22. Sound travels 1120 feet a second. How long will it take to travel 20,060 feet?

23. A contractor had 12 men hired at $3.25 a day, and 24 men at $2.50 a day. What did he pay them all in 15 days?

24. The longest side of a triangle is 56 inches long. The shortest side is 24 inches long. The other side is the average of these two. What is the perimeter of the triangle?

LESSON NINETY

TO MULTIPLY A FRACTION BY A FRACTION

I. EXPLANATION. Suppose a problem requiring the multiplication of $\frac{5}{6}$ by $\frac{3}{4}$ thus,

$$\frac{3}{4} \times \frac{5}{6}$$

Since $\frac{3}{4} = 3 \times \frac{1}{4}$, there are really two operations, namely, partition and multiplication.

$$\frac{1}{4} \text{ of } \frac{5}{6} = \frac{5}{24},$$

because the $\frac{5}{6}$ or $\frac{20}{24}$ is divided into 4 equal parts. One of these parts is $\frac{5}{24}$.

$$3 \times \frac{5}{24} = \frac{15}{24} \text{ or } \frac{5}{8}$$

Notice here three operations, as follows:

1st. The denominator (6) of the original multiplicand was multiplied by the denominator (4) of the original multiplier, giving the denominator (24) of the product.

2nd. The numerator (5) of the original multiplicand was multiplied by the numerator (3) of the original multiplier, giving the numerator of the product, (15).

3rd. The product ($\frac{15}{24}$) was reduced to its lowest terms.

The operation, therefore, becomes

$$\frac{3}{4} \times \frac{5}{6} = \frac{15}{24} \text{ or } \frac{5}{8}$$

The reduction of the product to its lowest terms can be performed by cancellation before the multiplication of the terms. Thus, in the above example, one of the numerators (3) and one of the denominators (6) are both divisible by the same number (3). If this operation is performed the operation becomes

$$\frac{\overset{1}{\cancel{3}}}{4} \times \frac{5}{\underset{2}{\cancel{6}}} = \frac{5}{8}$$

Sometimes considerable labor is saved by this means.

If either multiplier or multiplicand is a whole number, change it to a fraction by writing 1 for its denominator. Then proceed as before. Thus,

$$9 \times \frac{5}{6} \text{ becomes, } \frac{\overset{3}{\cancel{9}}}{1} \times \frac{5}{\underset{2}{\cancel{6}}} = \frac{15}{2} = 7\tfrac{1}{2}$$

Or,
$$\frac{3}{8} \times 12 \text{ becomes, } \frac{3}{\underset{2}{\cancel{8}}} \times \frac{\overset{3}{\cancel{12}}}{1} = \frac{9}{2} = 4\tfrac{1}{2}$$

II. DRILL.

Teacher will dictate problems, and drill upon the mechanical operation until it can be readily performed.

UNCLASSIFIED PROBLEMS

1. (a) $\frac{7}{8} \times \frac{24}{56} = ?$ (b) $12 \times \frac{5}{6} = ?$ (c) $\frac{5}{7} \times 28 = ?$

2. A man had 1824 dollars in the bank and 816 dollars in a safety box. He sold a house for 1754 dollars, and bought another for 2015 dollars. How much money had he then?

3. $407\frac{1}{6} + 79\frac{2}{3} - 397\frac{8}{9} = ?$

4. Divide \$215,688 into 256 equal parts. How many dollars are in each part?

5. (a) Change $107\frac{3}{17}$ to 17ths. (b) $\frac{274}{19} = ?$

6. A triangle has one side $15\frac{1}{9}$ yards long, another $7\frac{5}{6}$ yards long, and a third side $12\frac{2}{3}$ yards long. What is its perimeter?

7. A dealer bought $827\frac{2}{5}$ yards of carpet, and sold $279\frac{5}{8}$ yards. How many yards had he left on hand?

8. How many times can 136 be taken from 134,232?

9. $867\frac{5}{6} + 464\frac{2}{9} - 1294\frac{5}{12} = ?$

10. The three sides of a three-cornered field are respectively $99\frac{5}{8}$ yards, $107\frac{1}{8}$ yards, 145 yards. How many yards long is the fence which surrounds the field?

11. 12 lines each $\frac{2}{9}$ of an inch long would make, if put together, a line how many inches long?

12. How many times can 692 be taken from 350,152?

13. A boulevard around a city was in the form of a rectangle $9\frac{1}{2}$ miles long and 7 miles wide. How long was the boulevard? (Draw figure.)

14. $78\frac{3}{14} + 109\frac{5}{7} + 19\frac{5}{6} + 59\frac{1}{3} = ?$

15. The space between two lines on a sheet of writing paper is $\frac{2}{7}$ of an inch wide. How wide are 5 such spaces?

16. A rectangle is $\frac{3}{4}$ of a yard long, and $\frac{7}{15}$ of a yard wide. Its area is what part of a square yard?

17. At \$23.75 an acre, how many acres of land can be bought for \$997.50?

18. A man travels 27 miles a day. How far will he travel in 54 days?

19. A horse trader sold 56 horses for \$6400 and thereby gained \$912. What did he pay apiece for them?

20. A room is 18 feet long and 15 feet wide. What is the area of its floor?

21. Find the cost of 9 books at $\frac{7}{9}$ of a dollar each.

22. A man left \$9000 to be divided equally among his 5 children. How much money did 3 of the children together receive?

23. A has 78 sheep. B has $\frac{2}{3}$ as many sheep as A. How many sheep have both?

24. From a can containing $12\frac{5}{6}$ gallons of oil, $4\frac{1}{8}$ gallons were sold at one time and $5\frac{3}{4}$ gallons at another time. How much oil remained in the can?

25. Each of 18 boxes contains $\frac{7}{8}$ of a pound of candy. How much candy is there in all the boxes?

26. A dealer sells 12 blank books at $\frac{1}{3}$ of a dollar each. What did he receive for them?

27. Find the cost of 17 overcoats at \$19.47 each.

28. $101\frac{3}{5} + 99 + \frac{2}{3} + 1763\frac{1}{2} = ?$

CHAPTER VII

STEPS OF FIFTH YEAR

LESSON NINETY-ONE

TO MULTIPLY WHEN ONE OF THE FACTORS IS A WHOLE NUMBER AND THE OTHER A MIXED NUMBER

(a) *When the multiplicand is mixed.*

I. EXPLANATION. *Problem.* In each of 9 kegs there were $56\frac{3}{5}$ pounds of sugar. How much sugar in all the kegs?

Solution. Imagined counting gives,

$56\frac{3}{5}$ pounds in 1 keg.
$9 \times$
$5\frac{2}{5}$
504
$509\frac{2}{5}$ pounds in 9 kegs.

$9 \times \frac{3}{5} = \frac{27}{5}$ or $5\frac{2}{5}$. Write the partial product $5\frac{2}{5}$ as shown in the figure, so that the units 5 will be in the units' column, and the fraction $\frac{2}{5}$ in the fraction column.

$9 \times 56 = 504$. Write the partial product 504 so that 4 will be in the units' column, 0 in the tens' column, and 5 in hundreds' place.

Adding the partial products we find that $509\frac{2}{5}$ pounds of sugar were in the 9 kegs.

(b) *When the multiplier is mixed.*

I. EXPLANATION. Suppose a problem giving,

$$\begin{array}{r} 272 \\ 24\frac{3}{7} \\ \hline 116\frac{4}{7} \\ 1088 \\ 544 \\ \hline 6644\frac{4}{7} \end{array}$$

$\frac{3}{7} \times 272 = \frac{816}{7} = 116\frac{4}{7}$

1. Multiply 272 by $\frac{3}{7}$. The partial product is $116\frac{4}{7}$. Write this so that hundreds will stand under hundreds, tens under tens, units under units, and fractions under fractions.

2. Multiply 272 by 4 units, and write the first figure of the partial product under *units*.

3. Multiply 272 by 2 *tens*, and write the first figure of the partial product under *tens*.

4. Add the partial products.

II. DRILL.

The teacher will dictate abstract problems until pupils are familiar with the processes.

UNCLASSIFIED PROBLEMS

1. If I paid $\$3791\frac{5}{6}$ for a house and lot, what would be the cost of 15 pieces of property each of the same value?

2. Find the area of a rectangle 18 feet $\times$ $12\frac{3}{5}$ feet.

3. A dealer had 17 rolls of carpet, $53\frac{2}{3}$ yards in each roll. He sold $279\frac{5}{9}$ yards. How many yards remained?

4. How many times can 859 be taken from 22,334?

5. How many times can 785 be taken from 31,864?

6. (a) Reduce $129\frac{7}{16}$ to an improper fraction. (b) $\frac{487}{19}$ = ?

7. (a) $34 \times 518\frac{5}{9}$ = ? (b) $315\frac{3}{4} \times \$98.64$ = ?

8. A farm was divided into 9 tracts, each containing $12\frac{5}{7}$ acres. How many acres did the farm contain?

9. In a class are 10 pupils 9 years of age, and 10 pupils $9\frac{1}{2}$ years of age. What is the average age of the class?

10. How many times can 586 be taken from 367,583?

11. A rectangle is $39\frac{1}{3}$ inches long and $27\frac{5}{7}$ inches wide. How much longer is one side than the other?

12. A square flower bed was 16 feet long and 16 feet wide. Around its edge a box hedge was planted. How long was the hedge? (Make drawing.)

13. Draw a triangle.

Suppose its sides are $17\frac{3}{4}$ miles, $19\frac{7}{8}$ miles, and $30\frac{5}{6}$ miles.

Find how much shorter the longest side is than the sum of the other two sides.

14. Find the area of a rectangle $47\frac{2}{3}$ feet $\times$ 28 feet.

15. (a) $\frac{3}{4} \times \frac{7}{9} = ?$ (b) $\frac{5}{7} \times \frac{21}{40} = ?$ (c) $\frac{4}{5} \times \frac{15}{16} = ?$

16. A merchant paid his bookkeeper $\$45\frac{3}{4}$ a week, his stenographer $\$18\frac{1}{2}$ a week, and his salesman $\$27\frac{2}{5}$ a week. How much did he pay them all in 24 weeks?

17. A farm was divided into four fields. One contained $24\frac{5}{6}$ acres; another contained $19\frac{3}{8}$ acres; another $22\frac{7}{12}$ acres, and the fourth $15\frac{3}{16}$ acres. How many acres were there in the farm?

18. From a barrel containing $30\frac{1}{5}$ gallons of vinegar, $9\frac{7}{15}$ gallons were used. How many gallons of vinegar remained in the barrel?

19. The average population of the wards of a small city was 1895. The total population of the city was 17,055. Into how many wards was the city divided?

20. A storekeeper owes a farmer for 5 dozen eggs at 23 cents a dozen, 6 dozen eggs at 30 cents a dozen, and 4 dozen eggs at 35 cents a dozen. The farmer wishes to settle the bill by taking calico at 15 cents a yard. How many yards of calico should he receive?

LESSON NINETY-TWO

TO MULTIPLY WHEN BOTH FACTORS ARE MIXED NUMBERS

I. EXPLANATION. Suppose a problem requiring $5\frac{2}{3} \times 17\frac{1}{6}$.

In elementary arithmetic it is better to reduce the mixed numbers to improper fractions and find their product.

Thus,

$$5\tfrac{2}{3} = \tfrac{17}{3}. \qquad 17\tfrac{1}{6} = \tfrac{103}{6}.$$

$$\tfrac{17}{3} \times \tfrac{103}{6} = \tfrac{1751}{18}, \text{ or } 97\tfrac{5}{18}.$$

The mechanical operations may be preserved, as follows:

$$\begin{array}{r} 103 \\ 17 \\ \hline 721 \\ 103 \\ \hline 18)1751(97\frac{5}{18} \\ 162 \\ \hline 131 \\ 126 \\ \hline \frac{5}{18} \end{array}$$

II. DRILL.

Teacher should dictate abstract problems until pupils are familiar with the operation.

UNCLASSIFIED PROBLEMS

1. I counted the words on a page of a book and found that they averaged $8\frac{1}{3}$ words to a line. There are $19\frac{4}{5}$ lines on the page. How many words did I count?

2. A field of $19\frac{7}{9}$ acres yielded an average of 32 bushels to the acre. How much wheat did the field yield?

3. Each of 14 bolts of cloth contained $32\frac{7}{8}$ yards. How many yards were there in all the bolts?

4. $1796\frac{3}{8} + 8076\frac{5}{8} - 597\frac{5}{16} = ?$

5. Divide 987,645 by 4183.

6. A rectangle is $19\frac{5}{6}$ yards long and $16\frac{4}{9}$ yards wide. What is its perimeter? (Draw figure.)

7. A grocer bought $15\frac{3}{4}$ tubs of butter, each containing 24 pounds. How much butter did he buy?

8. Draw a triangle whose sides are 19 feet, $16\frac{1}{2}$ feet, and $14\frac{3}{4}$ feet. Find its perimeter.

9. 58 suits were sold at an average price of \$19.34. What was received for them?

10. Find $\frac{1}{29}$ of \$1741.16.

11. Draw a square whose side is $15\frac{3}{4}$ inches. Compute its area and its perimeter.

12. Find the area of a rectangle $19\frac{3}{7}$ inches by 14 inches.

13. How many times can 84 be taken from 34,104?

14. In each of 29 bins there were $137\frac{5}{9}$ bushels of oats. How many bushels of oats were in all the bins? How many bushels were there in 12 bins?

15. Find the area of a rectangle 27 inches by $9\frac{5}{7}$ inches.

16. Draw a rectangle $9\frac{1}{2}$ inches long and $5\frac{2}{3}$ inches wide. Compute its area.

17. Find the cost of $\frac{3}{4}$ of a yard of cloth at $\$\frac{5}{6}$ a yard.

18. A horse cost \$96. If it is sold for $\frac{7}{12}$ of cost, what is the loss?

19. If $9\frac{5}{6}$ inches are cut from a yardstick, how many inches long will the remainder be?

20. A boy had 4 gallons of milk to deliver to his customers. If each customer took 2 pints, how many customers were there?

21. In making a drawing, a boy drew a line $3\frac{7}{16}$ inches long. The line should have been only $2\frac{5}{8}$ inches long. How much must he erase?

LESSON NINETY-THREE

DIVISION OF FRACTIONS

(a) *When the dividend and divisor are both fractions.*

I. EXPLANATION. *Problem.* If cloth is worth $\$\frac{5}{6}$ a yard, what part of a yard can be bought for $\$\frac{3}{4}$?

Solution. Imagined counting gives, $\frac{5}{6})\frac{3}{4}$

This may be written $\frac{3}{4} \div \frac{5}{6} =$, in which $\frac{3}{4}$ is the dividend and $\frac{5}{6}$ the divisor.

Without attempting to give any reasons for the procedure, train pupils:

1st. To write the dividend.

2nd. To change the sign from $\div$ to $\times$.

3rd. To invert the divisor.

The numerical expression of this problem when so dealt with, becomes

$$\frac{3}{4} \times \frac{6}{5} =,$$

which pupils have already learned to solve as follows:

$$\frac{3}{4} \times \frac{6}{5} = \frac{18}{20}, \text{ or } \frac{9}{10}.$$

Therefore, $\frac{9}{10}$ of a yard of cloth can be purchased for $\$\frac{3}{4}$.

II. DRILL.

Teacher will dictate similar abstract problems, until pupils can perform the operation readily.

If either dividend or divisor is a whole number it can be changed to an improper fraction by writing 1 as the denominator.

Thus,

$$7 \div \frac{3}{4} = \frac{7}{1} \times \frac{4}{3} = \frac{28}{3} \text{ or } 9\frac{1}{3}.$$

$$\frac{5}{6} \div 15 = \frac{\overset{1}{\cancel{5}}}{6} \times \frac{1}{\underset{3}{\cancel{15}}} = \frac{1}{18}.$$

(b) *When the divisor is a mixed number and the dividend either a whole or mixed number.*

Both dividend and divisor are to be reduced to improper fractions, and the operation performed as explained at the beginning of this lesson. Thus:

(1) $4256 \div 78\frac{2}{3} = ?$

This becomes,

$$\frac{4256}{1} \div \frac{236}{3} = \frac{\overset{1064}{\cancel{4256}}}{1} \times \frac{3}{\underset{59}{\cancel{236}}} = \frac{3192}{59} = 54\tfrac{6}{59}.$$

(2) $7\frac{5}{6} \div 3\frac{4}{9}$

$$\frac{47}{6} \div \frac{31}{9} = \frac{47}{\cancel{6}_{2}} \times \frac{\overset{3}{\cancel{9}}}{31} = \frac{141}{62} = 2\tfrac{17}{62}.$$

(c) *When the dividend is a mixed number and the divisor a whole number.*

In this case time and labor may often be saved by not reducing to improper fractions, but by proceeding as follows:

Problem. Suppose a problem requiring $19{,}836\frac{2}{3}$ to be divided by 29.

1st. Divide 19,836 by 29, giving the quotient 684.

2nd. Divide $\frac{2}{3}$ by 29, giving the quotient $\frac{2}{87}$.

Then the entire quotient is $684\frac{2}{87}$.

The work may be written thus:

$$\begin{array}{r|l} 29 & 19836\frac{2}{3} \\ \hline & 684\frac{2}{87} \end{array}$$

$$\begin{array}{r} 174 \\ \hline 243 \\ 232 \\ \hline 116 \\ 116 \\ \hline \end{array}$$

$$\tfrac{2}{3} \div 29 = \tfrac{2}{3} \times \tfrac{1}{29} = \tfrac{2}{87}.$$

If the whole number of the dividend can not be exactly divided by the divisor, the undivided remainder will be a mixed number instead of a fraction, and must be reduced to an improper fraction before being divided by the divisor.

Thus:

$7)\underline{586\frac{4}{5}}$

83 and $5\frac{4}{5}$ over.

$5\frac{4}{5} = \frac{29}{5}$.

$\frac{29}{5} \div 7 = \frac{29}{5} \times \frac{1}{7} = \frac{29}{35}$.

$83 + \frac{29}{35} = 83\frac{29}{35}$, quotient.

Whether the quotient is a mixed number, or a whole number with a remainder consisting of a fraction or a mixed number, depends upon the nature of the problem.

Suppose in each of the above problems the dividend names the number of acres in a tract of land to be subdivided into smaller tracts, the size of which is shown in each instance by the divisor. Then the quotient will equal the number of these smaller tracts.

In such case the quotient in the first problem will not be $684\frac{2}{7}$, but 684, with a remainder of $\frac{2}{3}$ of an acre, *i.e.*, there will be 684 smaller tracts, with $\frac{2}{3}$ of an acre left undivided.

Likewise, the quotient in the second problem will not be $83\frac{29}{35}$, but 83, with a remainder of $5\frac{4}{5}$ acres, *i.e.*, there will be 83 smaller tracts, with $5\frac{4}{5}$ acres left undivided.

II. DRILL.

Teacher will dictate abstract problems in division of fractions until pupils are familiar with the operation.

UNCLASSIFIED PROBLEMS

1. How many times can $\frac{5}{9}$ be taken from $\frac{25}{27}$?

2. It required $150\frac{2}{5}$ yards of carpet to cover the floors of 8 rooms. What was the average number of yards to a room?

3. A farm of 429 acres is divided into tracts of $28\frac{3}{5}$ acres each. How many of these tracts are there?

4. A rectangle is $586\frac{3}{4}$ feet long and 98 feet wide. What is its area?

5. Draw a triangle whose sides are assumed to be $19\frac{3}{4}$ feet, $27\frac{5}{6}$ feet, and $14\frac{1}{2}$ feet. Find its perimeter.

6. What is the length of the iron fence around a square whose side is $207\frac{5}{6}$ feet?

7. (a) $36\frac{2}{3} \times 1083 = ?$ (b) $59\frac{1}{4} \times 2675 = ?$

8. Divide 987,645 by 4183.

9. Find the area of a rectangle $17\frac{1}{6}$ feet by $5\frac{2}{3}$ feet.

10. Draw a baseball diamond. It is a square, 90 feet on each side. If a player makes a home run, how far does he run? If he makes a two base hit, how far does he run?

11. (a) $35\frac{3}{10} \times 156 = ?$ (b) $63 \times 149\frac{7}{9} = ?$

12. $75\frac{3}{8} \div 24\frac{3}{7} = ?$

13. $197 \div 12\frac{4}{5} = ?$

14. An iron triangle had one side $14\frac{4}{5}$ inches long, one 12 inches long, and one $10\frac{5}{7}$ inches long. How long was the rod from which the triangle was made?

15. A grocer bought $15\frac{3}{4}$ tubs of butter, each containing 24 pounds. How much butter did he buy?

16. $87{,}935 \div 724 = ?$

17. A stock dealer had $\$1349\frac{5}{8}$ to invest in horses at \$125 each. How many horses could he buy, and how much money would remain unspent?

18. A jar contains $\frac{5}{6}$ of a pint of preserved strawberries. How many pints of strawberries will 24 such jars contain?

19. A dealer received \$39.28 for writing pads at 4 cents apiece. How many did he sell?

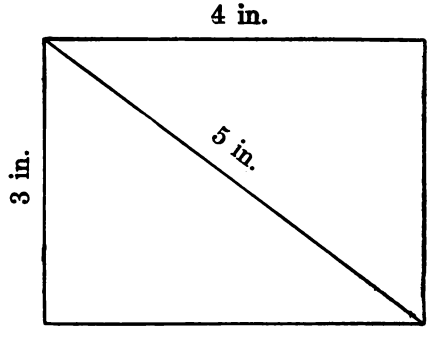
20. A horse cost \$154 and a cow $\frac{5}{7}$ as much. How many dollars did the horse cost more than the cow?

21. The front of a yard measures $51\frac{1}{2}$ feet. What did the sidewalk cost at \$2 a yard?

LESSON NINETY-FOUR

THE DIAGONALS OF A RECTANGLE

I. EXPLANATION. Draw a rectangle 3 inches wide and 4 inches long. Thus:



Draw a line from the upper left corner to the lower right corner. It is called a *diagonal* of the rectangle.

If a line were drawn from the lower left corner to the upper right corner, it also would be a diagonal.

The two diagonals of a rectangle are equal.

II. DRILL.

Teacher passes pointer along upper side from right to left; pupils write, "4 inches."

Teacher continues along diagonal to lower right corner; pupils write, "5 inches."

Teacher continues along lower side to lower left corner; pupils write, "4 inches."

Teacher asks, "Over how many inches has the pointer passed?"

Pupils add and answer, "13 inches."

UNCLASSIFIED PROBLEMS

1. Draw a rectangle $22\frac{1}{2}$ feet wide and 30 feet long. From the upper left to the lower right corner draw a diagonal. It will be $37\frac{1}{2}$ feet long. If a boy walks along the diagonal from the upper left to the lower right corner, then to the lower left corner, and then to the upper left corner, how many feet will he walk altogether?

2. A rectangle is 37 feet long and $27\frac{3}{4}$ feet wide. Its diagonal is $46\frac{1}{4}$ feet. How much longer is the sum of the length and breadth than the diagonal?

3. $67\frac{2}{5} \div 9\frac{3}{4} = ?$

4. $287\frac{2}{5} \div 19 = ?$

5. How many times can 3586 be taken from 743,528?

6. Draw a rectangle $37\frac{1}{3}$ yards long and 28 yards wide. Draw both diagonals. Each of them will be $46\frac{2}{3}$ feet long. Find the total length of the six lines you have drawn.

7. A triangular flower bed whose sides are respectively $18\frac{5}{7}$ feet, $16\frac{4}{5}$ feet, and $19\frac{6}{35}$ feet, is surrounded by a cement curb. How long is the curb?

8. A picture, with its frame, is 24 inches long and 14 inches wide. How many square inches of wall space does it cover?

9. A man wished to put $178\frac{3}{5}$ quarts of vinegar into jugs. Each of the jugs held 8 quarts. How many jugs would be filled, and how many quarts of vinegar would be left over?

10. A rectangle is $28\frac{5}{7}$ feet long and 14 feet wide. What is its area?

11. A rectangle is 23 inches wide and $30\frac{2}{3}$ inches long. Each of its diagonals is $38\frac{1}{3}$ inches. At each of its corners a tack is driven part way into the board on which the rectangle is drawn. If a string is fastened to the upper left tack, then drawn taut around the upper right, the lower left, the lower right, and back to the upper left tack, how much string is used? (Draw figure.)

12. Draw the baseball diamond. A square whose side is 90 feet, has a diagonal of 127 feet.

How far must the catcher throw the ball when a runner tries to steal second? How far when a runner tries to steal third?

13. The fence on one side of a square field is 278 feet long. How long is the entire fence around the field?

14. Find the area of a rectangle whose length is $49\frac{2}{3}$ miles and whose width is $28\frac{4}{5}$ miles.

15. $415\frac{5}{9} \div \frac{2}{3} = ?$

16. Find the perimeter of a rectangle $29\frac{3}{4}$ feet by $17\frac{5}{6}$ feet.

17. Mr. Thomas invested \$217,943 in city houses. A number of houses were bought at an average price of \$2842, when it was found that not enough money was left to buy another house at the average price. How many houses had he bought, and how much money was left?

18. 6 bushels of oats were put into sacks, each sack holding $\frac{2}{3}$ of a bushel. How many sacks were used?

19. A room is 21 feet wide. It is to be covered with carpet $\frac{7}{8}$ of a yard wide. How many strips of carpet will be needed?

20. I spent $\$49\frac{1}{6}$ for coal at $\$9\frac{5}{6}$ a ton. How many tons did I buy?

21. Find the cost of $19\frac{3}{4}$ cords of wood, if 1 cord cost \$6.40.

22. How many square feet of ground in a lot $98\frac{2}{3}$ feet long and $36\frac{1}{2}$ feet wide?

23. If a brick is 8 inches long and 4 inches wide, how many bricks will it take to cover a square foot? (Make a drawing to show how the bricks are placed.)

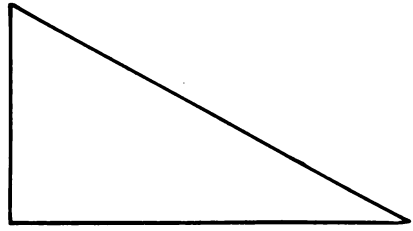
24. William had \$500. He spent $\frac{3}{4}$ of it for a team of horses. How many dollars had he left?

25. The perimeter of a triangle is 45 feet. One side is $18\frac{2}{3}$ feet long; another side is $12\frac{5}{6}$ feet long. How long is the third side?

LESSON NINETY-FIVE

THE RIGHT-ANGLED TRIANGLE

I. EXPLANATION AND DRILL. Draw a triangle with one square corner. Thus:



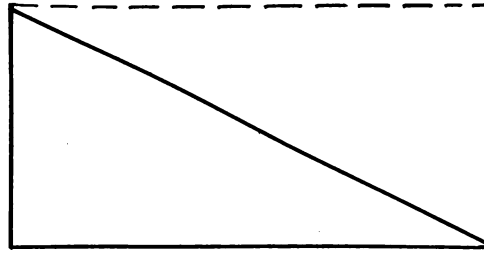
A square corner is called a *right angle.*

A triangle with a square corner is called a *right-angled triangle.*

Try to draw a triangle with two square corners. How many right angles can a triangle have?

The long side of a right-angled triangle is called the *hypotenuse.*

The other two sides are called *legs.*



Notice that a right-angled triangle is one-half of a rectangle.

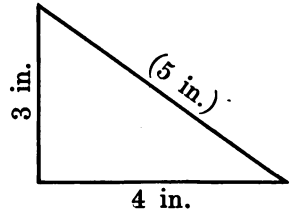
The hypotenuse of the triangle is the diagonal of the rectangle.

The legs of the triangle are the length and width of the rectangle.

If two sides of a right-angled triangle are known, the third side can be found by computation. This computation is too difficult for an elementary arithmetic.

There is, however, one type of a right-angled triangle occurring frequently in practice whose sides are in the proportion of 3, 4, and 5. Triangles of this type can be easily solved.

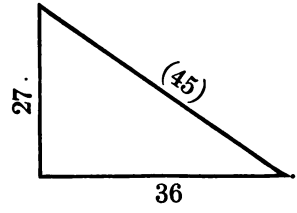
Draw a right-angled triangle whose legs are respectively 3 inches, and 4 inches. By measurement you will find its hypotenuse to be 5 inches. Thus:



If the legs are doubled, that is, made 6 and 8, the hypotenuse will be doubled, that is, made 10.

So, if the legs are made three times as long as in the type, the hypotenuse will be made three times as long, etc.

Take this triangle:

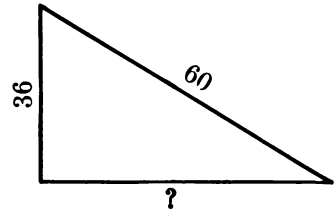


What is the hypotenuse?

$$27 \div 3 = 9$$
$$36 \div 4 = 9$$

The legs are 9 times those of the type. Hence the hypotenuse is $9 \times 5 = 45$.

Take this triangle:

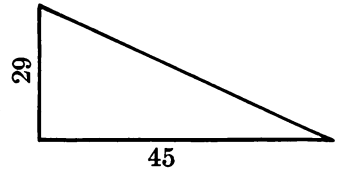


$60 \div 5 = 12$
$36 \div 3 = 12$

The two sides given are 12 times as large as the corresponding sides of the type. Hence the longer leg is $12 \times 4 = 48$.

Notice that we must always have two sides given, in order to determine whether the given triangle is similar to the type.

Try this:



$45 \div 5 = 9$
$29 \div 3 = 9\frac{2}{3}$

The triangle is not similar to the type, and can not be solved by this method.

UNCLASSIFIED PROBLEMS

1. The two legs of a right-angled triangle are 42 feet and 56 feet. How long is its hypotenuse?

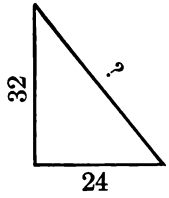
2. $16\frac{4}{5} \div 29\frac{7}{15} = ?$

3. $6794 \div 59\frac{2}{5} = ?$

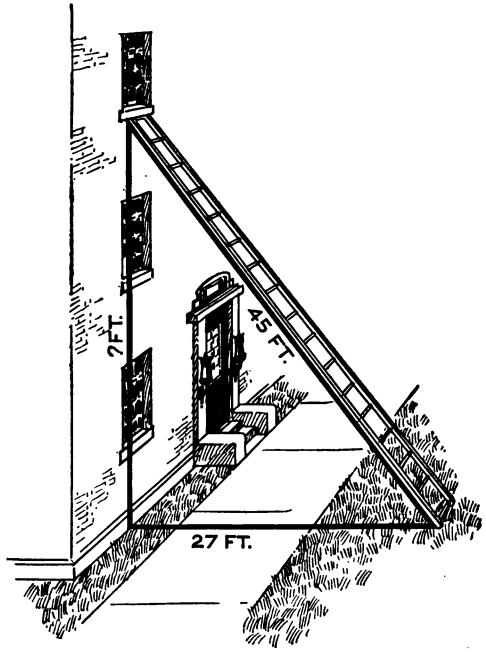
4. $8647\frac{5}{7} \div 9 = ?$

5. Find the area of a rectangle 107 feet wide and 2076 feet long.

6. Find whether or not this right-angled triangle is similar to the type whose sides are 3, 4, and 5. If it is, find its hypotenuse and perimeter.



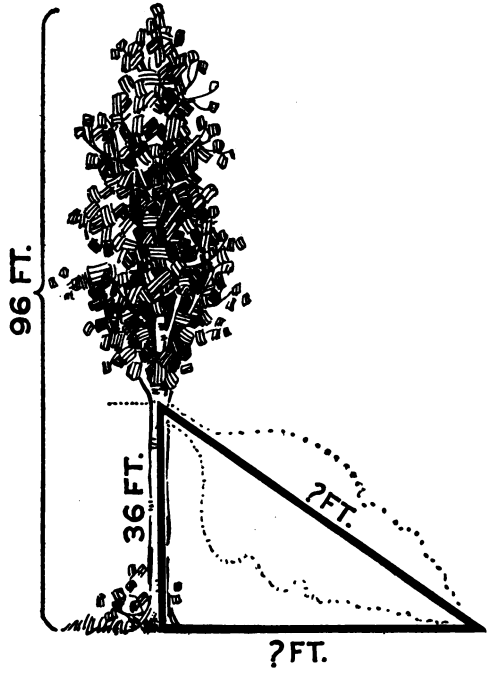
7. One end of a ladder 45 feet long rested against a win-



dow-sill. The other end rested on the ground 27 feet from the house. How high was the window-sill from the ground?

8. A rectangle 34 inches long and $25\frac{1}{2}$ inches wide has diagonals $42\frac{1}{2}$ inches long. What is the total length of the four sides and the two diagonals?

9. Find the area of a rectangle $19\frac{4}{5}$ feet by $7\frac{3}{7}$ feet.



10. A tree 96 feet high was broken by a storm 36 feet above the ground. The upper part bent over until the top just touched the ground. How far was it from the foot of the tree to the spot where the top touched the ground?

11. Find $\frac{1}{35}$ of 26,437.

12. The hypotenuse of a right-angled triangle is $48\frac{3}{4}$ feet. Its shorter leg is $29\frac{1}{4}$ feet. How long is the other leg?

LESSON NINETY-SIX

DECIMAL FRACTIONS

Fractions which have 10, 100, etc., for denominators are called *decimal* fractions. In this book only those decimal fractions are used which have either 10 or 100 for a denominator.

$\frac{2}{10}$ may be written .2, in which 2 expresses the numerator, and the denominator 10 is expressed by the fact that the numerator has only one place to the right of the decimal point. $\frac{2}{100}$ may be written .02, in which 2 expresses the numerator, and the denominator 100 is expressed by the fact that the numerator has two places to the right of the decimal point.

Read the following. Name the numerator; tell how it is expressed.

Name the denominator; tell how it is expressed.

.85	.4	.04	.25	.09	.9	3.6
3.04	.56	.08	.7	2.7	5.16	4.17

The first place to the right of the decimal point is called tenths; the second is called hundredths.

We have already seen that 85¢ may be written \$.85. This is because 1 cent is $\frac{1}{100}$ of a dollar; hence, 85 cents are $\frac{85}{100}$ of a dollar, or \$.85. So, \$.09 is $\frac{9}{100}$ of a dollar, and \$2.16 are $2\frac{16}{100}$ dollars.

ADDITION AND SUBTRACTION

To add or subtract decimals, we arrange the numbers so that the decimal points are in a column.

Then add or subtract as in whole numbers. In the result place a decimal point under the column of decimal points, thus:

(1)	(2)	(3)	(4)	(5)
5.85	6.85	7.13	6.75	2.1
4.6	.79	−2.09	− .98	−1.59
.78	.4	5.04	5.77	.51
+ .05	+1.03			
11.28	9.07			

Notice that when one of the numbers has fewer decimal places than the others, we may annex a cipher, or suppose one to be annexed.

Find the sums:

(1)	(2)	(3)	(4)	(5)	(6)
302.97	4.9	56.97	5.8	684.8	417.08
4.8	63.17	14.85	13.97	5.18	74.9
29.06	2.18	196.4	8.07	17.48	6.83
.08	5.07	87.04	.7	6.04	17.24
1.97	209.18	1.57	.07	54.17	86.9
+ 18.65	+ 59.6	+ .6	+24.76	+ 27.7	+ 9.45

(7) 8.5 + .94 + 16.42 = ?
(8) 903.75 + 86.9 + .07 = ?
(9) 2.94 + .86 + 19.4 = ?

Find the remainders:

(1)	(2)	(3)	(4)	(5)	(6)
684.97	5.06	84.6	268.14	95.4	106.
− 29.89	− .97	− 9.87	−237.6	−89.35	− 9.65

UNCLASSIFIED PROBLEMS

1. A rectangle is 13.35 inches long and 9.7 inches wide. What is its perimeter?

2. If cloth is worth $\$\frac{7}{8}$ a yard, what part of a yard can be bought for $\$\frac{1}{2}$?

3. The perimeter of a triangle is 51.64 yards. One side is 15.6 yards and another side 14.96 yards. How long is the third side?

4. A rectangle is $43\frac{2}{3}$ feet by $17\frac{1}{5}$ feet. Find its area.

5. A farmer raised $1254\frac{2}{3}$ bushels of oats. He sold them in 4 equal lots. How many bushels of oats in each lot?

6. A city reservoir has a walk around the top. If the reservoir is a square each side of which is $109\frac{4}{5}$ yards, how long is the walk?

7. $118\frac{3}{5} \div 9\frac{7}{10} = ?$

8. $5964 \div 5\frac{1}{4} = ?$

9. The shorter leg of a right-angled triangle is $26\frac{1}{4}$ inches. Its hypotenuse is $43\frac{3}{4}$ inches. What is the length of its longer leg?

10. The hypotenuse of a right-angled triangle is $41\frac{1}{4}$ feet, and its shorter leg is $24\frac{3}{4}$ feet. How long is the other leg?

11. The side of a square is $9\frac{7}{8}$ feet. What is its area?

12. $3578\frac{2}{3} \div 46 = ?$

13. In a miner's cabin, 246 ounces of gold dust were hidden in old tomato cans. Each can contained $17\frac{4}{7}$ ounces. How many cans were used?

14. A rectangle is 5.9 inches long and 3.25 inches wide. What is its perimeter?

15. 111 barrels of flour worth \$6.35 a barrel were traded for 37 barrels of sugar. What was the sugar worth a barrel?

16. 29 shirts were sold for \$39.15. What was the average price received?

17. 18 horses were bought at \$110 each. They were sold at a gain of \$108. For how much were they sold?

18. $4\frac{2}{3}$ yards of cloth cost $\$9\frac{1}{3}$. What will $9\frac{1}{2}$ yards cost at the same price?

19. The sides of a triangle are 14 feet, 9.6 feet, and 8.25 feet. What is the perimeter of the triangle?

LESSON NINETY-SEVEN

TO MULTIPLY WHEN ONE OR BOTH FACTORS HAVE DECIMALS

Multiply as in whole numbers, and point off in the product as many decimal places as there are in both factors together.

Thus:

(1)	(2)	(3)	(4)
1.87 ft.	15.6	124	1.25
9 ×	1.7 ×	.6 ×	4.3 ×
16.83	10 92	74.4	375
	15 6		5 00
	26.52		5.375

Notice in (1) there are two decimal places in the multiplicand and none in the multiplier, making two in the product.

In (2) there is one decimal place in each factor, making two in the product.

In (3) there is one decimal place in the multiplier and none in the multiplicand, making one in the product.

In (4) there are two decimal places in the multiplicand and one in the multiplier, making three in the product.

Point off the following products:

(1) 6 × 5.7 = 342
(2) 5.6 × 28.4 = 15904
(3) 56 × 2.43 = 13608
(4) 2.13 × 86 = 18318
(5) 3.4 × 7.28 = 24752
(6) 2.8 × 15.6 = 4368
(7) 8 × 14.2 = 1136
(8) .7 × 1.84 = 1288
(9) 1.9 × 5.7 = 1083
(10) 26 × 2.34 = 6084

Find the products:

(1)	(2)	(3)	(4)
7.97	89.6	80.6	9.13
93 ×	4.6 ×	5.7 ×	60.7 ×

(5)	(6)	(7)	(8)
28.46	812	580	5.17
7 ×	.7 ×	.24 ×	83 ×

UNCLASSIFIED PROBLEMS

1. Find the area of a rectangle 10.8 feet × 4.6 feet.

2. The side of a square is 18.41 yards. What is its perimeter?

3. A man earns $3.25 in one day. What does he earn in 17 days?

4. What is the perimeter of a rectangle $19\frac{3}{4}$ inches by $12\frac{5}{8}$ inches?

5. $198\frac{5}{8}$ acres are divided into 6 equal tracts. How many acres are in each tract?

6. Find the perimeter of a triangle whose sides are 17.45 inches, 19.3 inches, and 8.23 inches.

7. 2.79 acres of land were sold for $325 an acre. How much was received for them?

8. From a tract of 4.9 acres, 3.25 acres were sold. How much of the tract was unsold?

9. $49\frac{3}{4} \div 8\frac{3}{7} = ?$

10. $17\frac{2}{5} \times 118\frac{1}{6} = ?$

11. $104 was received for umbrellas sold at $3.25 each. How many umbrellas were sold?

12. A yard is 125.4 feet long and 42.67 feet wide. How much greater is the length of the yard than its width?

13. How much will 129 ounces of candy sell for at 6 cents an ounce?

LESSON NINETY-EIGHT

DIVISION OF DECIMALS

I. EXPLANATION.

(1)

62)94.86(1.53
62
328
310
186
186

(2)

48)993.6(20.7
96
336
336

Notice that in (1) and (2) there are no decimal places in the divisor, and that the quotient has the same number of decimal places as the dividend.

When this is the case, we can locate the decimal point of the quotient with great ease and certainty by writing the quotient above the dividend, taking care that the first quotient figure is written directly above the right hand figure of the first partial dividend. The decimal points of dividend and quotient will then be in column. Thus:

(2) becomes,

20.7
48)993.6
96
336
336

It is desirable, therefore, that the divisor always be a whole number. If it contains a decimal it can be made an integer by multiplying it by 10, in case there is one decimal place; or by multiplying by 100, in case there are two decimal places.

Thus:

(3)		(4)
96.4)559.12(	becomes,	5.8 964)5591.2 4820 771 2 771 2

Notice that in (3) there is one decimal place in the divisor. We dispose of it by moving the decimal point one place to the right, (*i.e.*, by multiplying the divisor by 10); but we must make a like change in the position of the decimal point of the dividend. This gives us (4), which is solved exactly like (1) and (2).

In following the rule, sometimes it is necessary to annex one or two ciphers to the dividend, in order to have places enough.

Thus:

(5)		(6)
3.48)52.2(	becomes,	15. 348)5220. 348 1740 1740

Also,

(7)		(8)
3.48)522(	becomes,	150. 348)52200. 348 1740 1740 0

Usually the dividend does not exactly contain the divisor. In such cases, after making the divisor a whole number and making a like change in the dividend, we must see that the

dividend contains two decimal places, annexing one or two ciphers, if necessary.

Take the following:

(9)

6.4)98.4(

becomes,

(10)

15.37

64)984.00

64

344

320

240

192

480

448

+

Notice that in changing (9) to (10), we first moved both decimal points one place to the right, in order to make the divisor a whole number. We then added two decimal places to the dividend by annexing two ciphers, in order to have two decimal places in the quotient.

(11)

.64)98.4(

becomes,

(12)

153.75

64)9840.00

64

344

320

240

192

480

448

320

320

We changed (11) to (12) by moving both decimal points two places to the right, in order to make the divisor a whole number. In doing this we annex one cipher to the right of the dividend. We then annexed two more ciphers to the right of the decimal point in the dividend, in order to have two decimal places in the quotient.

(13)

.64)984(

becomes,

(14)

1537.5

64)98400.0Ø

64

344

320

240

192

480

448

320

320

In (14) the division became exact when we used the first decimal cipher of the dividend. The second cipher was, therefore, not needed.

II. DRILL.

Arrange the following in proper form for division. Put the quotient figures where they belong, and locate the decimal points in the quotients:

1. $23.94 \div .57 = 42$
2. $56.94 \div 7.8 = 7.3$
3. $34.5 \div .46 = 75$
4. $34.6 \div 25 = 1.38$
5. $8.36 \div 7.1 = 1.17$
6. $26.24 \div 8.2 = 3.2$

7. 24.3432 ÷ .42 = 5 7 9 6
8. .192 ÷ .96 = 2
9. .0102 ÷ .34 = 3
10. 1.8 ÷ .56 = 3 2 1

In the following, arrange for division, move the decimal point and divide:

1. 584 ÷ 1.6 = ?
2. 47.2 ÷ 3.6 = ?
3. 5.09 ÷ 2.6 = ?
4. 2875.4 ÷ 9 = ?
5. 486.7 ÷ .5 = ?
6. .07 ÷ .34 = ?
7. 1.4 ÷ 70 = ?
8. 54.6 ÷ .25 = ?
9. 2.75 ÷ 2.5 = ?

UNCLASSIFIED PROBLEMS

1. 584 yards of cloth were purchased for the sewing teacher. A piece 1.6 yards long was cut off for each pupil. How many pupils were supplied, if all the cloth was used?

2. A rectangle is 7.9 inches long. Its perimeter is 22.8 inches. How wide is it?

3. At $2 a bushel what is $\frac{4}{5}$ of a bushel of apples worth?

4. A lady bought $19\frac{1}{8}$ yards of silk at $\$2\frac{1}{3}$ a yard. What did it cost her?

5. What are 23.4 tons of hay worth at $16.8 a ton?

6. 7 spelling books cost $\frac{14}{15}$ of a dollar. What did one book cost?

7. $5\frac{1}{4}$ yards of calico were bought for $8\frac{1}{2}$ cents a yard. What was paid for the calico?

8. A merchant bought silk at $1.98 a yard and sold it at $2.35 a yard. How much did he gain on 128 yards?

LESSON NINETY-NINE

BILLS AND ACCOUNTS

BILLS.

When goods are bought at a store without being paid for at the time, they are *charged* to the person buying them.

All the goods charged to any person are written down together in a book.

This is called an *account.*

A *bill* is a statement of an account, and is usualiy written out and sent to the person who has bought the goods and owes the money.

Portland, Sept. 10, 1916.

Mr. William Jones,

Bought of George Ford

1916						
Aug.	3	$1\frac{1}{2}$ lb. cornstarch,	@	12¢	$	18
"	5	2 lb. coffee,	@	38¢		76
"	8	3 lb. butter,	@	41¢	1	23
					$ 2	17

Received Payment,
George Ford.
Oct. 1, 1916.

The above bill consists of three items, namely, $1\frac{1}{2}$ lb. of cornstarch, bought August 3rd, at 12 cents a pound; 2 lb. of coffee, bought August 5th, at 38 cents a pound; and 3 lb. of butter, bought August 8th, at 41 cents a pound.

To *extend* the items of a bill is to find the amount of each of them by multiplication. Thus the extensions in the above bill are made by (1) multiplying 12¢ by $1\frac{1}{2}$, and obtaining $.18 as the amount of the first item; (2) by taking $2 \times 38¢ = \$.76$,

as the amount of the second item; (3) by taking $3 \times 41¢ = \$1.23$, as the amount of the third item.

To *foot* a bill, we add the amounts of the various items. Thus the above bill is footed by adding \$.18, \$.76, and \$1.23, obtaining \$2.17.

To receipt a bill, that is, to show that it has been paid, the person to whom the money is owed writes the words, "Received Payment," giving the date on which payment was made, and signs it with his own name. See page 343.

The pupil will (1) copy, (2) extend, (3) foot, and (4) receipt the following bills:

Chicago, Feb. 4, 1914.

Mr. James Wilson,

Bought of Henry Fuller

1914						
Jan.	7	$3\frac{1}{2}$ lb. butter,	@	38¢		
"	10	4 doz. eggs,	@	40¢		

Cleveland, March 3, 1917.

Mr. Henry Wells,

Bought of William Miller

1917						
Feb.	9	5 cans soup,	@	25¢		
		4 lb. sugar,	@	10¢		
"	15	1 doz. oranges,	@	45¢		
		2 lb. raisins,	@	35¢		

Pittsburgh, April 10, 1915.

Mr. William Roberts,

Bought of Frederick Hansom

1912						
Mar.	4	2½ lb. lamb chop,	@	18¢		
"	9	3 lb. roast beef,	@	25¢		
"	15	2½ lb. fish,	@	20¢		
"	20	2½ lb. chicken,	@	28¢		

Salem, Oct. 3, 1918.

Mr. Henry Rice,

Bought of Chas. Jenkins

1918						
Sept.	3	4½ yd. of silk,	@	$1.50		
"	9	3 yd. of ribbon,	@	45¢		
"	17	2 doz. buttons,	@	25¢		
"	26	3 yd. of linen,	@	65¢		

Oakland, April 3, 1916.

Mr. William Richmond,

Bought of Henry James

1916						
Mar.	16	2 advanced geographies,	@	$1.20		
		2 arithmetics,	@	65¢		
		4 blank books,	@	15¢		

Make out bills for the following sales. Assume that you are the seller. Invent names for the buyers, and dates for the times of purchase. Extend, foot, and receipt the bills:

1. 2 lb. cheese @ 14¢; 1 head lettuce @ 5¢; 2½ lb. breakfast food @ 10¢.

2. 1 lb. cocoa @ 59¢; 2 lb. prunes @ 9¢; 2 pt. syrup @ 26¢; ½ lb. mustard @ 30¢.

3. 3½ yd. satin @ $1.80; 3 yd. lace @ 90¢; 4 yd. cloth @ 80¢; 5 yd. braid @ 15¢.

4. ½ gal. olive oil @ $3.00; 2 qt. currants @ 9¢; 1 box apples @ $2.25; 1 lb. tea @ 75¢.

5. 2 pairs shoes @ $4.25; 1 pair rubbers @ 85¢; 1 pair slippers @ $1.75.

6. 9 spools of thread @ 5¢; 4½ yd. canvas @ 12¢; 5 yd. braid @ 17¢.

7. 2 lb. dried apricots @ 25¢; 2 pt. catsup @ 18¢; 9 cakes of soap @ 9¢.

8. 2½ yd. silk @ $2.00; 4½ yd. ribbon @ 60¢; 3 doz. buttons @ 40¢.

9. 3 lb. steak @ 25¢; 2 lb. chops @ 18¢; 4 lb. chicken @ 26¢.

10. 2 grammars @ 65¢; 3 histories @ 90¢; 4 blank books @ 8¢.

11. 1 gal. olive oil @ $2.90; 2 lb. coffee @ 41¢; 3 doz. eggs @ 35¢.

12. 2 lb. cornstarch @ 12½¢; 5 lb. sugar @ 10¢; 2 doz. oranges @ 42¢.

13. 3 lb. raisins @ 36¢; 2 pt. catsup @ 15¢; ½ lb. cocoa @ 62¢.

14. 1½ lb. breakfast food @ 12¢; 3 lb. dried prunes @ 12¢; 2 pt. syrup @ 24¢.

15. 2 lb. chocolate @ 48¢; 4 lb. dried apricots @ 26¢; 3 qts. currants @ 12¢.

16. 3 yd. lace @ 75¢; 5 yd. calico @ 15¢; 8 yd. linen @ 65¢.

17. 2 pairs fine shoes @ $6.00; 1 pair slippers @ $2.75; repairing 80¢.

18. 2 lb. steak @ 26¢; 3 lb. fish @ 19¢; 3 lb. roast beef @ 24¢.

19. 3 lb. mutton chops @ 17¢; 5 lb. chicken @ 22¢; 2 lb. bacon @ 19¢; 4 lb. fish @ 20¢.

LESSON ONE HUNDRED

REDUCTION OF DENOMINATE NUMBERS

(a) *To a lower unit.*

I. EXPLANATION. *Problem.* 4 ft. = how many inches?

Solution. 1 ft. = 12 in.

4 ft. = 4 × 12 in. = 48 in.

Problem. A string is 4 ft. 9 in. long. How many inches in the string?

Solution. 1 ft. = 12 in.

4 ft. = 4 × 12 in. = 48 in.

48 in. + 9 in. = 57 in., length of string.

For convenience the work may be arranged as follows:

ft.	in.
4 ×	9
12 in.	
48 in.	
+ 9 in.	
57 in.	

II. DRILL.

(1) How many oz. in 14 lb. 12 oz.?

(2) How many qt. in 8 gal.?

(3) How many qt. in 8 gal. 2 qt.?

(4) How many hours in 15 da. 6 hr.?

(5) How many inches in 4 yd. 2 ft. 9 in.?

(b) *To a higher unit.*

I. EXPLANATION. *Problem.* 9 feet are how many yards?

Solution. There are 3 ft. in 1 yard. The number of yards in 9 ft. equals the number of times 3 ft. can be taken from 9 ft. or 3. For convenience this may be written,

3 ft.)9 ft.

3 = no. of yd.

Problem. How many yd. in 11 ft.?

Solution. The number of yd. equals the number of times 3 ft. (1 yd.) can be taken from 11 ft. or 3 times, and 2 ft. over. Hence, 3 yd. 2 ft.

For convenience this may be written,

$$3 \text{ ft.})\underline{11 \text{ ft.}}$$

3 and 2 over = 3 yd. 2 ft.

II. DRILL.

(1) How many gal. in 10 qt.?

(2) How many days in 125 hr.?

(3) How many bu. in 29 pk.?

(4) Reduce 573 in. to yd., ft., and in.

NOTE. First reduce to ft., the remainder being in. Then reduce the ft. to yd., the remainder being ft.

(5) Reduce 182 oz. to lb. and oz.

UNCLASSIFIED PROBLEMS

1. A rectangle is 4.6 ft. long $\times$ 9.3 ft. What is its area?

2. A rectangle is 32 in. $\times$ 24 in. Express its area in sq. ft. and sq. in.

3. The string on one kite is 80 ft. long. The string of another kite is 62 ft. long. Express the total length of the two strings in yd. and ft.

4. The shorter leg of a right-angled triangle is 28.86 inches. The other leg is 38.48 inches. How long is the hypotenuse?

5. How many pt. in 3 gal., 2 qt., 1 pt.?

6. Express 3192 min. as da., hr., min.

7. The sides of a triangle are 9.4 ft., 8.37 ft., and 12.4 ft. What is its perimeter?

8. Express 2 tons, 98 lb., as lb.

9. Express 42 oz. as lb. and oz.

10. 17 dollars were spent for cloth at \$1.80 a yard. How many yards was bought?

LESSON ONE HUNDRED ONE

ADDITION AND SUBTRACTION OF DENOMINATE NUMBERS

(a) *Addition.*

I. EXPLANATION. *Problem.* The fence on the front of a corner lot is 16 yd. 2 ft. 9 in. The fence on the side is 32 yd. 2 ft. 4 in. How long are both fences?

The solution is obviously by addition.

Write the numbers so that like units will be in the same column; thus:

yd.	ft.	in.
16	2	9
32	2	4
49	2	1

3)5(1 yd. 12)13(1 ft.
3 12
2 ft. 1 in.

Adding the inches we have 13 inches, or 1 ft. 1 in.

Place the 1 in. under the column of inches, and carry 1 ft. to the column of feet.

Adding the ft. we have 5 ft., or 1 yd. 2 ft.

Place the 2 ft. under the column of feet, and carry the 1 yd. to the column of yards.

Adding the yards we have 47 yards.

The sum is 49 yd., 2 ft., 1 in.

(b) *Subtraction.*

I. EXPLANATION. *Problem.* From a barrel of cider containing 28 gal. 2 qt. 1 pt., 9 gal. 3 qt. 1 pt. were used. How much cider remained in the barrel?

The operation is obviously subtraction.

Write the numbers so that like units will be in the same column, thus:

gal.	qt.	pt.
28	2	1
− 9	3	1
18	3	0

1 pt. from 1 pt. leaves 0.

3 qt. from 2 qt. can not be taken.

Take 1 of the 28 gal. and add it to the 2 qt. making 6 qt. 3 qt. from 6 qt. leaves 3 qt.

We have already taken 1 gal. from the 28 gal., and have 9 more to take, making 10 gal. from 28 gal., which leaves 18 gal.

II. DRILL.

(1) Add 5 lb. 9 oz.; 16 lb. 8 oz.; 24 lb. 7 oz.

(2) From 18 lb. 5 oz. take 9 lb. 12 oz.

(3) From 3 da. 4 hr. 7 min. take 14 hr. 52 min.

(4) Add 1 bu. 2 pk.; 9 bu. 3 pk.; 3 bu. 1 pk.

(5) From 9 gal. 2 qt. take 4 gal. 3 qt. 1 pt.

(6) Add 4 sq. yd. 6 sq. ft. 109 sq. in.; 9 sq. yd. 4 sq. ft. 97 sq. in.; and 2 sq. yd. 5 sq. ft. 25 sq. in.

(7) Find the sum of,

17 gal. 2 qt.; 8 gal. 3 qt. 1 pt.; 9 gal. 1 qt. 1 pt.

UNCLASSIFIED PROBLEMS

1. $9.5 \div 1.25 = ?$

2. Reduce 4 mi. 13 yd. to yards.

3. Express 5299 yd. as mi. and yd.

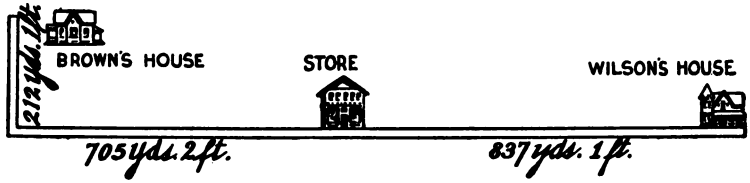
4. What was received for 5 gal. 3 qt. of milk at 7 cents a quart?

5. If a cord of wood is worth $6.75, how much is $\frac{3}{8}$ of a cord worth?

6. The side of a square is 16.8 yd. What is its perimeter? What is its area?

7. A rectangle is 19 in. $\times$ $14\frac{1}{4}$ in. Find its perimeter.

8. Mr. Wilson's house is how much nearer the store than Mr. Brown's house?



9. What was received for 4192 neckties at 98¢ apiece?

10. Find the perimeter of a rectangle 18.4 in. $\times$ 15.23 in.

11. One cask contains 9 gal. 3 qt. of vinegar, another contains 7 gal. 2 qt., a third contains 5 gal. 1 qt. 1 pt. How much vinegar is there in the three casks?

12. $15 \times 187\frac{3}{4} = ?$

13. $87 \div 5\frac{1}{6} = ?$

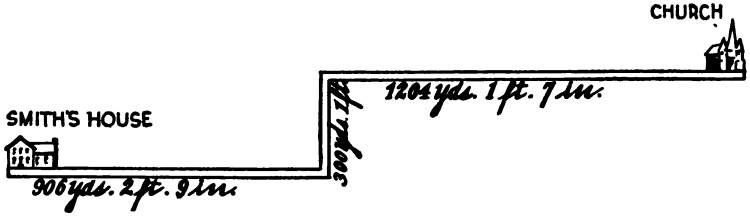
14. $137\frac{2}{5} \div 96\frac{3}{8} = ?$

15. A bag of coffee contained 49 lb. 9 oz. If 32 lb. 12 oz. were sold, how much coffee remained in the bag?

16. A rectangle is $18\frac{2}{3}$ ft. by $12\frac{7}{8}$ ft. What is its area?

17. $193\frac{4}{5} \div 12\frac{7}{8} = ?$

18. How far is it from Mr. Smith's house to the church?



19. An automobile running at the rate of $34\frac{2}{3}$ miles an hour ran from Roswell to Blanton in $2\frac{1}{4}$ hours. How far is it from Roswell to Blanton?

LESSON ONE HUNDRED TWO

MULTIPLICATION AND DIVISION OF DENOMINATE NUMBERS

(a) *Multiplication.*

I. EXPLANATION. *Problem.* 3 bags of coffee weigh 39 lb. 9 oz. each. What do they all weigh?

The operation is obviously multiplication.

Solution.

$$\begin{array}{rr} 39 \text{ lb.} & 9 \text{ oz.} \\ & 3 \times \\ \hline 118 & 11 \\ \hline \end{array}$$

$$16)27 \text{ oz.} (1 \text{ lb.}$$
$$\underline{16}$$
$$11 \text{ oz.}$$

3×9 oz. $= 27$ oz. or 1 lb. 11 oz.

Write the 11 oz. under oz., and carry the 1 lb.

3×39 lb. $= 117$ lb., $+ 1$ lb. carried, make 118 lb.

(b) *To divide a denominate number into equal parts.*

Problem. 3 equal bags of coffee weigh altogether 118 lb. 11 oz. What do they weigh apiece?

$\frac{1}{3}$ of 118 lb. 11 oz. = ?

$$\begin{array}{r} 9 \\ \hline 28 \\ 27 \\ \hline \end{array}$$

1 lb. = 16 oz.

$$\begin{array}{r} 16 \text{ oz.} \\ 11 \text{ oz.} \\ \hline \end{array}$$

$\frac{1}{3}$ of 27 oz. = 9 oz.

$$\begin{array}{r} 27 \\ \hline \end{array}$$

That is, $\frac{1}{3}$ of 118 lb. 11 oz. = 39 lb. 9 oz.

We first take $\frac{1}{3}$ of 118 lb., and find it to be 39 lb. and 1 lb. over. Reducing the 1 lb. over to oz. we find it is 16 oz., to which adding 11 oz. we have 27 oz.

$\frac{1}{3}$ of 27 oz. = 9 oz.

II. DRILL.

(1) Find 3 × 5 yd. 2 ft. 9 in.
(2) Find 9 × 15 gal. 3 qt. 1 pt.
(3) Find $\frac{1}{4}$ of 19 days 8 hr.
(4) Find $\frac{1}{12}$ of 1807 lb. 12 oz.
(5) Find $\frac{1}{5}$ of 48 gal. 3 qt.
(6) Find 7 × 15 hr. 24 min.

UNCLASSIFIED PROBLEMS

1. A rectangle is 7 yd. 2 ft. long, and 4 yd. 1 ft. wide. How many square feet does it contain?

2. The butter in one tub weighed 16 lb. 9 oz.; that in another tub, 14 lb. 3 oz.; that in a third tub, 19 lb. 4 oz. What was the weight of the butter in the three tubs?

3. What is the area of a rectangle whose width is $\frac{3}{5}$ of a foot and length $1\frac{1}{9}$ feet?

4. What is the perimeter of a right-angled triangle whose legs are respectively 16.8 inches and 22.4 inches?

5. A line which should have been $6\frac{5}{8}$ inches long was made $7\frac{1}{4}$ inches long. How long is the part of the line which must be erased?

6. What is the cost of 135 lb. of coffee at 23 cents a pound?

7. A railroad train runs 34.6 miles an hour. How far will it run in 18.9 hours?

8. 3 bags of coffee contained 127 lb. 8 oz. What was the average weight of the bags?

9. A flower bed is $16\frac{3}{4}$ ft. long and $6\frac{8}{9}$ ft. wide. How many square feet does it contain?

LESSON ONE HUNDRED THREE

UNCLASSIFIED PROBLEMS

1. One barrel contained 28 gal. 2 qt. of vinegar, another 27 gal. 3 qt., another 26 gal. 1 qt. What was the average quantity to a barrel?

2. A man built a fence in three days. The first day he built 19.4 ft. of the fence, the second day 21.3 ft., the third day 22.9 ft. How long was the fence?

3. The side of a square is $15\frac{3}{4}$ ft. Find its perimeter.

4. Four houses had extension wires attached to their telephones. The first house had 3 ft. 1 in. of wire attached, the second had 3 ft. 6 in., the third 4 ft. 5 in., and the fourth 2 ft. 5 in. How many feet and inches of wire were used in making the extensions?

5. A rectangle is 14.78 in. $\times$ 10.3 in. Find its perimeter.

6. The load carried by each soldier in a regiment was 29 pounds. If the soldiers of the regiment carried altogether 14,674 pounds, how many men were in the regiment?

7. Three bins contain respectively $48\frac{3}{4}$ bu. of potatoes, $56\frac{3}{8}$ bu., and $47\frac{5}{12}$ bu. If the potatoes had been divided equally among the three bins, what would each contain?

8. The side of a square is 7.26 inches. Find its perimeter.

9. A party of workmen built a piece of road 215 ft. long in 6 days. How many feet and inches did they average a day?

10. 354 shirts were sold for \$346.92. What was the average price?

11. An iron bar that should have been $35\frac{1}{2}$ in. long, was made $41\frac{1}{4}$ in. long. How much must be cut off?

12. Make out a bill for the following purchases. Make the extensions and footing, and receipt the bill. Write your own name as purchaser and invent a name for the seller:

2 gal. olives at \$3.09 a gallon; 12 cakes of soap at 9 cents a cake; 3 boxes of apples at \$2.14 a box; 4 pounds of coffee at 38 cents a pound.

13. A girl spent each day, 1 hr. 19 min. in studying her geography, and 55 min. in studying her arithmetic. In 5 days how much time did she spend on these two studies?

14. The hypotenuse of a right-angled triangle is 320 ft. and its longer leg is 256 feet. Find its perimeter.

15. Find the area of a rectangle $9\frac{3}{8}$ ft. $\times$ $6\frac{2}{9}$ ft.

16. A footpath led across three fields. Across the first field the path was 107 yd. 2 ft. long; across the second field 94 yd. 1 ft.; across the third field 78 yd. 2 ft. How long was the path?

17. The three sides of a triangular boulevard are 987 yards, 1234 yards, and 1463 yards. How long is the boulevard?

18. How many times can 3174 be taken from 2,247,192?

19. A rectangle is 7.4 feet long and 5.3 feet wide. Find its perimeter.

20. An automobile ran 259.88 mi. in 7.3 hr. What was its average speed an hour?

21. What was the cost of 129 boxes of soap at \$3.15 a box?

22. A man had a farm of 108.4 acres. He added to it at one time 27.3 acres, and at another time 35.54 acres. He then sold 42.29 acres. How many acres had he remaining?

23. A farmer raised 7000 bushels of wheat one year, 8500 bushels the next, and 7900 bushels the next. How many bushels a year did the 3 crops average?

24. A lady used $2\frac{3}{4}$ yards of silk to make a waist. How many waists could she make of 33 yards?

25. A rectangle is 30 ft. 8 in. long, 23 ft. wide, and its diagonal is 38 ft. 4 in. The length and width together are how much longer than the diagonal?

26. A ribbon 82.8 inches long can be cut into how many pieces each 4.6 inches long?

LESSON ONE HUNDRED FOUR

UNCLASSIFIED PROBLEMS

1. Each of 5 ladies had in her purse $9\frac{4}{5}$ dollars. Each of them spent $4\frac{9}{10}$ dollars. How much money remained in the 5 purses?

2. The side of a square is $16\frac{2}{7}$ inches. Find its area.

3. From a bolt of cloth containing $36\frac{1}{4}$ yards, $19\frac{1}{2}$ yards were sold. How much cloth remained unsold?

4. Make out a receipted bill for the following purchases; John Wyman is the purchaser, and William Hitchcock the seller:

5 yards of flannel at 85 cents a yard; 3 yards of satin at $1.75 a yard; 4 spools of thread at 5 cents a spool; 5 yards of canvas at 14 cents a yard.

5. A store sold shirts worth $284.17, shoes worth $194.32, and hats worth $185.36. Of this $325.87 was charged. How much cash was received from the sales?

6. A rectangle is 13.8 in. × 6.9 in. Find its area.

7. Our house tank holds 821 gal. 2 qt. of water. The tank on the house next to us holds 737 gal. 3 qt. How much more water does our tank hold than our neighbor's?

8. The side of a square is 19.8 feet. Find its area.

9. The hypotenuse of a right-angled triangle is 82 inches, and its longer leg 65.6 inches. How long is its shorter leg?

10. Five hats were sold for $4.80, $6.15, $5.24, $3.81, $4.25, respectively. What was the average price?

11. Find the perimeter of a rectangle $10\frac{3}{4}$ in. × $8\frac{5}{6}$ in.

12. 21 qt. of ice cream were bought for a banquet. Only 16 qt. 1 pt. were used. How much was wasted?

13. A barrel of vinegar holds 32 gallons. How many barrels will be needed to hold 4704 gallons?

14. The perimeter of a rectangle is 20 ft. Its length is 6 ft. Find its width. Draw figure.

LESSON ONE HUNDRED FIVE

UNCLASSIFIED PROBLEMS

1. 249 neckties were sold at an average price of $.76. How much was received for them?

2. 15 jars of candy contained 2 lb. 4 oz. each. If 9 oz. were sold from each, how much candy was left in the 15 jars?

3. 294 acres of land produced 10,584 bushels of wheat. What was the average yield to the acre?

4. A rectangle is 29 ft. 7 in. long and 14 ft. 8 in. wide. Find its perimeter.

5. In a harbor 15 ships were anchored. The length of their anchor chains averaged 694 ft. 9 in. What was the total length of these chains?

6. The side of a square is 18 yards and 2 feet. Find its perimeter.

7. How many times can 26 be taken from 130,234?

8. A rectangle is $\frac{2}{5}$ of an inch long, and $\frac{3}{10}$ of an inch wide. Find its perimeter.

9. In a cellar were three bins. One held 28 bu. 3 pk. of potatoes, one 32 bu., and the third 29 bu. 2 pk. What quantity of potatoes was in the cellar?

10. The legs of a right-angled triangle are $29\frac{1}{4}$ in. and 39 in. Find its hypotenuse.

11. A party of three boys went fishing. One had a line 21 ft. 7 in. long. Another had a line 18 ft. 4 in. long. The third line was 15 ft. 4 in. long. What was the average length of the lines?

12. Make out a receipted bill for the following purchases; William Maston is the buyer, and Samuel Harmon the seller:

5 pounds of steak at 25 cents a pound; 3 pounds of fish at 18 cents a pound; 4 pounds of roast beef at 28 cents a pound; 3 pints of catsup at 17 cents a pint.

13. 22 lb. 3 oz. of flour were used in making 5 cakes of the same size. How many lb. and oz. of flour were used for each cake?

14. The sides of a triangle are 8.3 feet, 4.75 feet, and 7 feet. Find the perimeter.

15. \$1919.52 was received from the sale of men's hats at an average price of \$3.87. How many were sold?

16. A rectangle is $\frac{2}{9}$ of an inch wide and $\frac{3}{8}$ of an inch long. Find its area.

17. A triangular field has one side $208\frac{1}{4}$ ft. long, another side $157\frac{2}{3}$ ft. long, and a third side $264\frac{5}{6}$ ft. long. A fence has been built around all of the field except $59\frac{5}{6}$ ft. How long is the fence?

18. \$558.72 was received from the sale of handkerchiefs at 18 cents apiece. How many were sold?

19. The sides of a triangle are $19\frac{2}{3}$ in., $14\frac{3}{5}$ in., and $7\frac{5}{6}$ in. Find its perimeter.

20. A ball of twine was originally 68 ft. 9 in. long. It is now 20 ft. 10 in. long. How much twine has been used?

21. 118 neckties were bought at 98 cents each and sold for \$1.25 each. What was the profit on them?

22. For a new school-room, the school board bought 42 desks at \$3.50 each; one teacher's desk at \$12.80; 4 chairs at \$1.45 each; one clock at \$5.80; pictures for \$8.75. What was spent on the room?

23. A man earns \$27 a week and spends \$18.87. How much can he save in 15 weeks?

24. A boy picked 17 quarts of raspberries on Monday, 19 quarts on Tuesday, 22 quarts on Wednesday, and 15 quarts on Thursday. His mother used 29 quarts, and the boy sold the remainder at 7 cents a quart. What did he receive for them?

25. A Marathon race is 26 mi. 385 yd. long. After a runner has covered 17 mi. 51 yd. 2 ft., how far is he from the finish?

TABLES AND FORMS

ADDITION TABLE

1	1	1	1	1	1	1	1	1
+1	+2	+3	+4	+5	+6	+7	+8	+9
2	3	4	5	6	7	8	9	10
2	2	2	2	2	2	2	2	
+2	+3	+4	+5	+6	+7	+8	+9	
4	5	6	7	8	9	10	11	
3	4	5	6	7	8	9		
+3	+4	+5	+6	+7	+8	+9		
6	8	10	12	14	16	18		
4	5	6	7	8	9			
+3	+4	+5	+6	+7	+8			
7	9	11	13	15	17			
9	9	9	9	9				
+3	+4	+5	+6	+7				
12	13	14	15	16				
8	8	8	8					
+3	+4	+5	+6					
11	12	13	14					
7	7	7						
+3	+4	+5						
10	11	12						
5	6	6						
+3	+3	+4						
8	9	10						

MULTIPLICATION TABLE

		1	2	3	4	5	6	7	8	9	10	11	12
½ of	2 ×	2	4	6	8	10	12	14	16	18	20	22	24
⅓ of	3 ×	3	6	9	12	15	18	21	24	27	30	33	36
¼ of	4 ×	4	8	12	16	20	24	28	32	36	40	44	48
⅕ of	5 ×	5	10	15	20	25	30	35	40	45	50	55	60
⅙ of	6 ×	6	12	18	24	30	36	42	48	54	60	66	72
⅐ of	7 ×	7	14	21	28	35	42	49	56	63	70	77	84
⅛ of	8 ×	8	16	24	32	40	48	56	64	72	80	88	96
⅑ of	9 ×	9	18	27	36	45	54	63	72	81	90	99	108
⅒ of	10 ×	10	20	30	40	50	60	70	80	90	100	110	120
1/11 of	11 ×	11	22	33	44	55	66	77	88	99	110	121	132
1/12 of	12 ×	12	24	36	48	60	72	84	96	108	120	132	144

TABLES OF DENOMINATE NUMBERS

1. CAPACITY

(a) LIQUID
2 pints (pt.) = 1 quart (qt.)
4 quarts (qt.) = 1 gallon (gal.)

(b) DRY
8 qt. = 1 peck (pk.)
4 pk. = 1 bushel (bu.)

2. WEIGHT

16 ounces (oz.) = 1 pound (lb.)
2000 lb. = 1 ton (T.)

3. TIME

60 minutes (min.) = 1 hour (hr.)
24 hr. = 1 day (da.)
7 da. = 1 week (wk.)
28, 30, or 31 da. = 1 month (mo.)
12 mo. = 1 year (yr.)
365 da. = 1 yr.

4. LINEAR

12 inches (in.) = 1 foot (ft.)
3 feet (ft.) = 1 yard (yd.)
1760 yd. = 1 mile (mi.)

5. SURFACE

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 sq. ft. = 1 square yard (sq. yd.)

6. VALUE

10 cents (¢) = 1 dime
10 dimes, 100¢ = 1 dollar ($)
$10 = 1 eagle

100¢ = $1 is the relation most used.

7. MISCELLANEOUS

12 single things = 1 dozen
20 single things = 1 score
24 sheets of paper = 1 quire
20 quires = 1 ream

FORMS OF ANALYSIS

1. ADDITION

Problem. John had 6 pencils and Mary had 4. How many had both?

Numerical Solution. (Determined by imagining the mode of counting):

6 pencils, John had
+4 pencils, Mary had
10 pencils, both had

Oral Analysis. They both had 6 pencils plus 4 pencils, or 10 pencils.

2. SUBTRACTION

Problem. Henry had 43 peaches and ate 9; how many had he left?

Numerical Solution. (Determined by imagining the mode of counting):

43 peaches, Henry had
−9 peaches, he ate
34 peaches, he had left

Oral Analysis. He had left 43 peaches minus 9 peaches, or 34 peaches.

3. MULTIPLICATION

Problem. Mary had 4 pincushions, with 134 pins in each cushion; how many pins were there in all?

Numerical Solution. (Determined by imagining the mode of counting):

134 pins, in each cushion
×4 = number of cushions
536 pins in all

Oral Analysis. There were 4 times 134 pins, or 536 pins.

4. PARTITION

Problem. 72 books of the same kind cost 288 dollars; how much was that apiece?

Numerical Solution. (Determined by imagining the mode of counting): $\frac{1}{72}$ of \$288 = \$4 apiece.

Oral Analysis. Each book cost one-seventy-second of 288 dollars, or 4 dollars

5. MEASUREMENT

Problem. A lady divided 8 cakes among her children, giving each 2, how many children had she?

Numerical Solution. (Determined by imagining the mode of counting):

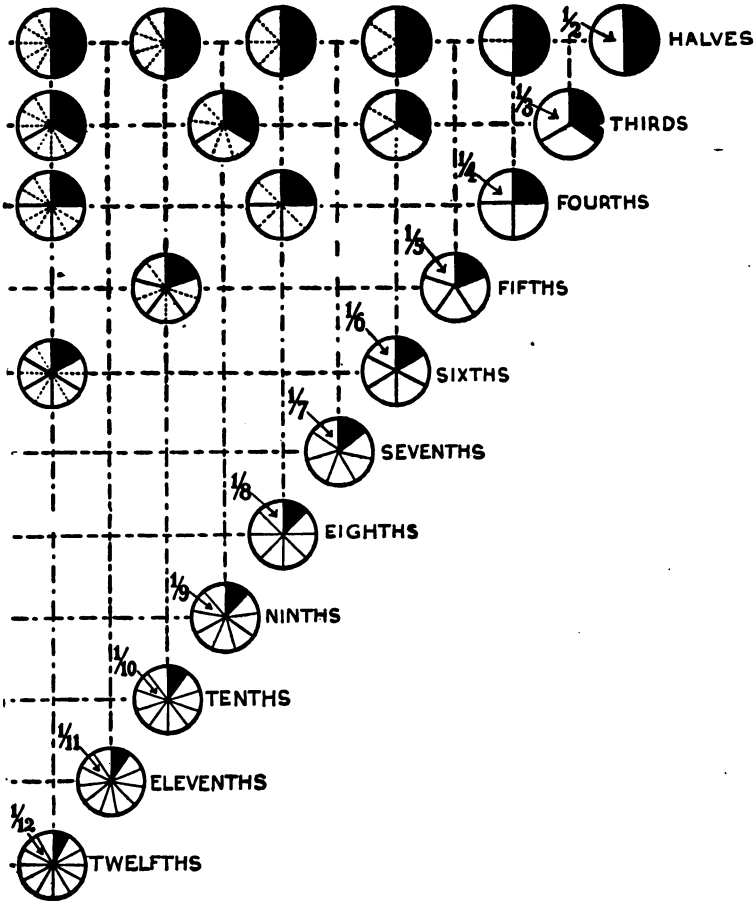
2 c.)8 cakes

4 = number of children.

Oral Analysis. The number of children equals the number of times 2 cakes can be taken from 8 cakes, or 4.

NOTE TO TEACHER. There has been more or less controversy over the two cases of division, *partition* and *measurement.* Some contend that there is but one kind of division, but generally when such an assertion is made, it is meant that there is but one computative device called division. This is true, but there are two situations which call for the employment of that device. In either case a plurality is to be divided into equal parts. In (1) the number of the parts is known, and their common value is to be found. In (2) the common value of the parts is known, and their number is to be found.

If division is considered as the reverse of multiplication, then in (1) the product and multiplier are known; in (2) the product and multiplicand are known.

Every reputable writer recognizes these two cases in some form or other, and if computation is to be derived from counting, their recognition is unavoidable.

REDUCTION TABLE OF FRACTIONS